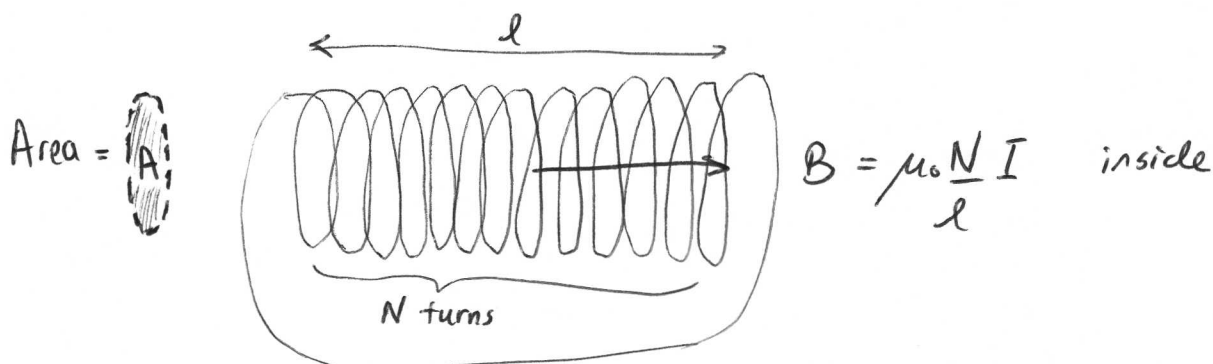


Continuing with inductance of a loop

$$\mathcal{E}MF = -\frac{d\Phi}{dt} = -L \frac{dI}{dt}$$

True if the only source of B-field is coming from the current circulating in the loop.

Example: Loop is a solenoid



$$\begin{aligned} \mathcal{E}MF &= \oint_{\text{along wire path}} \vec{E} \cdot d\vec{l} \\ &= N \int (\nabla \times \vec{E}) \cdot d\vec{a} \\ &= N \int \frac{d\vec{B}}{dt} \cdot d\vec{a} \\ &= NA \frac{N}{l} \frac{dI}{dt} \end{aligned}$$

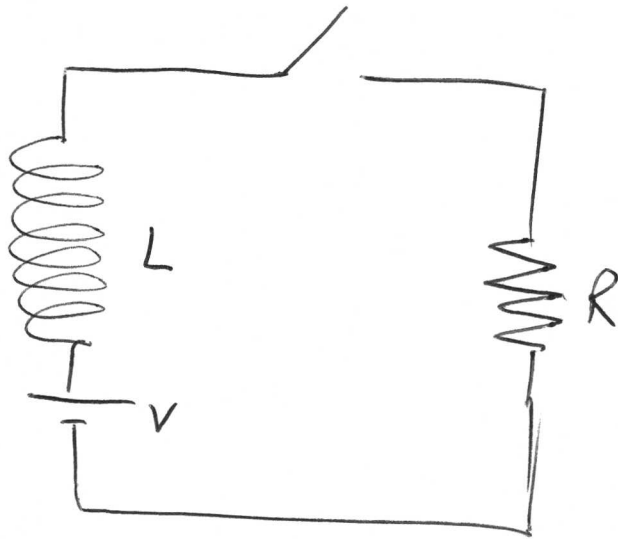
$$\Rightarrow \text{Inductance } L = \frac{AN^2}{l}$$

(2)

One consequence of inductance:

you cannot turn on a B-field instantly

Example

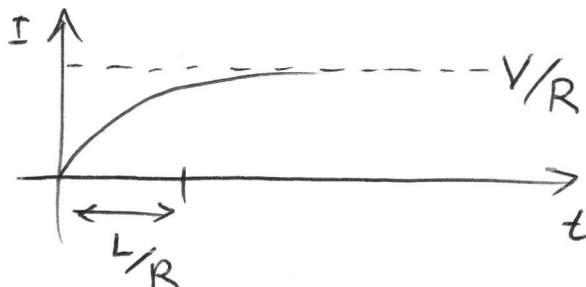


when switch is closed, net voltages around loop is zero.

$$V - L \frac{dI}{dt} - IR = 0$$

Solve this differential equation with $I=0$ at $t=0$.

$$\Rightarrow I(t) = \frac{V}{R} (1 - e^{-\frac{R}{L}t})$$



(3)

Another consequence of inductance is that it takes work to turn on a \vec{B} -field:

$$\frac{dW}{dt} = IV \quad \left(\text{pushing current against the EMF} \right)$$

$$= I \left(L \frac{dI}{dt} \right)$$

Total work done

$$\int_{t=0}^{t_f} \frac{dW}{dt} dt = L \int I \frac{dI}{dt} dt$$

$$= L \int_0^{I_f} I dI$$

$$= \frac{1}{2} L I_f^2$$

↙ final current.

This energy is not dissipated like I^2R in a resistor, the energy is stored.

Where is it stored? In the motion of the charge?
In the magnetic field?
Can't distinguish.

④

An alternative way to calculate the stored energy

is

$$W = \frac{1}{2\mu_0} \int_{\text{all space}} |\vec{B}|^2 d^3\vec{r}$$

POP QUIZ

⑤
DAY 13

In class we showed that $W = \frac{1}{2} L I^2$ for a loop of inductance L .

If the loop was a very long solenoid, show that

$$\frac{1}{2} L I^2 \approx \frac{1}{2\mu_0} \int_{\text{volume inside solenoid}} |\vec{B}|^2 d^3r$$