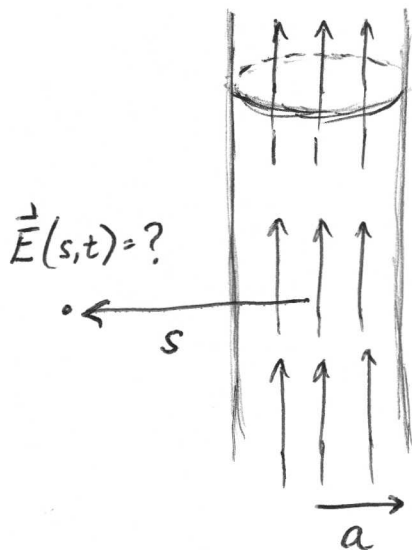


## Calculating the $\vec{E}$ -field generated by $\frac{\partial \vec{B}}{\partial t}$

Consider a cylindrical region of non-zero  $\vec{B}$ -field that changes over time.



inside the cylindrical region

$$\vec{B} = B_0 e^{-\alpha t} \hat{z}$$

time dependence.

outside

$$\vec{B} = 0$$

Find  $\vec{E}(s,t)$  at a distance  $s$  from the symmetry axis.

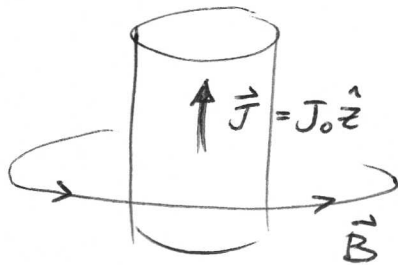
Solution: Use symm arguments and stokes's theorem to solve the differential eqn  $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ .

$$-\frac{\partial \vec{B}}{\partial t} = \begin{cases} \alpha B_0 e^{-\alpha t} \hat{z} & s \leq a \\ 0 & s > a \end{cases}$$

This vector-field "generates" an  $\vec{E}$  field, analogous to the ~~same~~ way that  $\vec{J}$  "generates" a  $\vec{B}$  field.

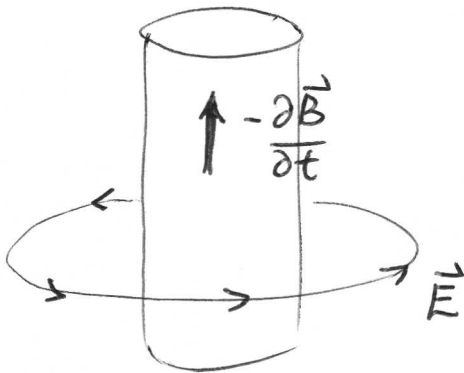
②

Recall how  $\vec{J}$  generates a  $\vec{B}$ -field in a geometry like this



$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

In this particular problem we have



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

The mathematical machinery of solving this differential eqn will be identical to problems involving  $\vec{B}$  generated by  $\vec{J}$ .

$$\oint_{\text{loop radius } s} \vec{E} \cdot d\vec{l} = \int_{\text{area inside loop}} \nabla \times \vec{E} \cdot d\vec{a}$$

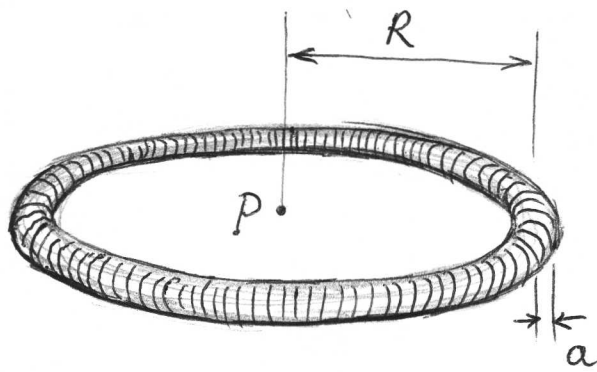
$$2\pi s E = \alpha B_0 e^{-\alpha t} \pi a^2$$

③

$$E = \frac{\alpha B_0 e^{-\alpha t} a^2}{2s}$$

$$\vec{E}(s, t) = \frac{\alpha B_0 e^{-\alpha t} a^2}{2s} \hat{\phi}$$

Pop Quiz



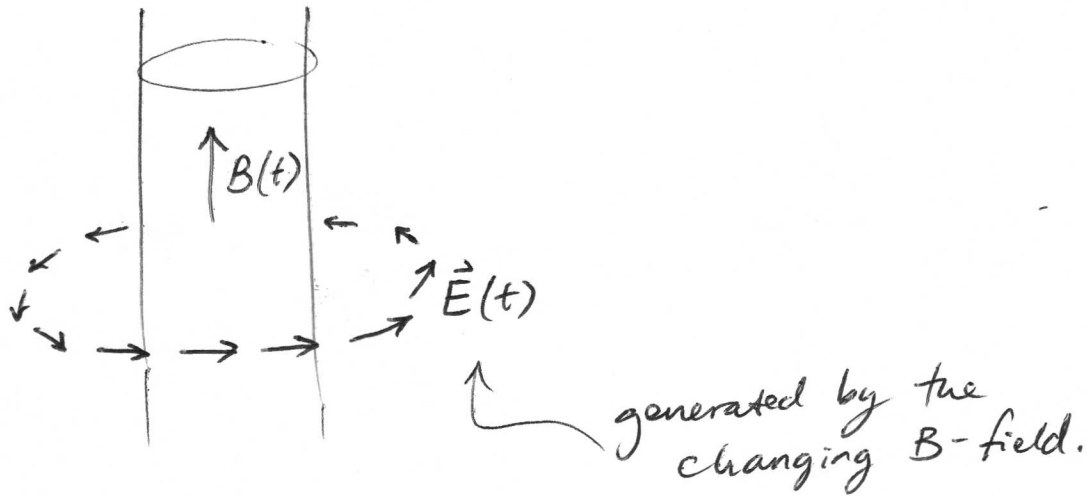
A toroidal solenoid with  $n$  turns per unit length has a small radius  $a$  and a large radius  $R \gg a$ .

The current in the solenoid varies with time,  $I = I_0 e^{-\alpha t}$ .

Find  $\vec{E}(t)$  in the center of the solenoid.

(4)

Return now to the example problem

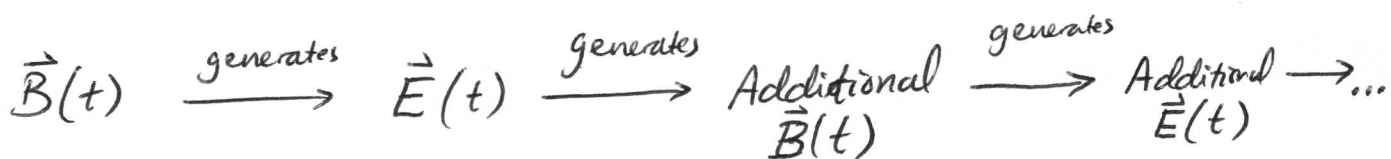


The changing  $\vec{E}$ -field will also "generate" something.

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

This term is a source of  $\vec{B}$ -field, on equal footing with  $\vec{J}$ .

To summarize



where will the calculation stop?!

In practice, we seldom go beyond  $\vec{B}(t) \xrightarrow{\text{generates}} \vec{E}(t)$ .  
Additional corrections to  $\vec{B}$  &  $\vec{E}$  are typically tiny.