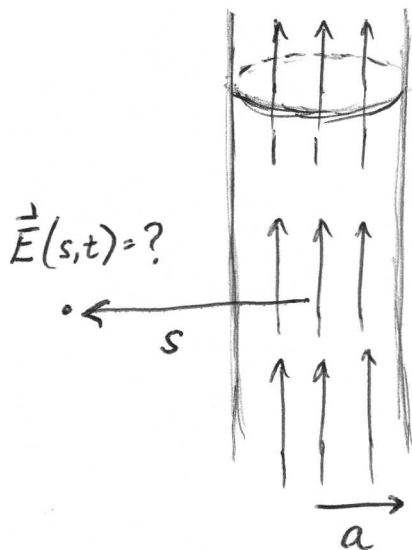


Calculating the \vec{E} -field generated by $\frac{\partial \vec{B}}{\partial t}$

Consider a cylindrical region of non-zero \vec{B} -field that changes over time.



inside the cylindrical region

$$\vec{B} = B_0 e^{-\alpha t} \hat{z}$$

time dependence.

outside

$$\vec{B} = 0$$

Find $\vec{E}(s,t)$ at a distance s from the symmetry axis.

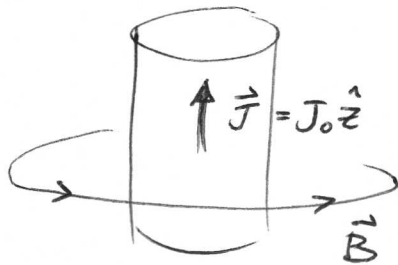
Solution: Use symm arguments and stokes's theorem to solve the differential eqn $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

$$-\frac{\partial \vec{B}}{\partial t} = \begin{cases} \alpha B_0 e^{-\alpha t} \hat{z} & s \leq a \\ 0 & s > a \end{cases}$$

This vector-field "generates" an \vec{E} field, analogous to the ~~same~~ way that \vec{J} "generates" a \vec{B} field.

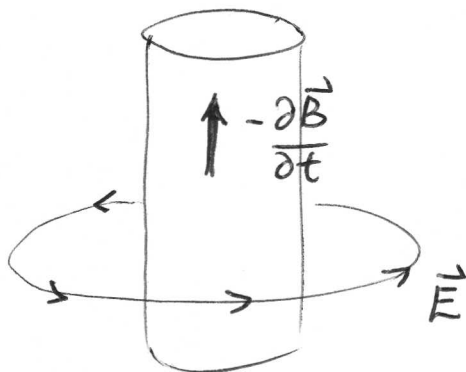
②

Recall how \vec{J} generates a \vec{B} -field in a geometry like this



$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

In this particular problem we have



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

The mathematical machinery of solving this differential eqn will be identical to problems involving \vec{B} generated by \vec{J} .

$$\oint_{\text{loop radius } s} \vec{E} \cdot d\vec{l} = \int_{\text{area inside loop}} \nabla \times \vec{E} \cdot d\vec{a}$$

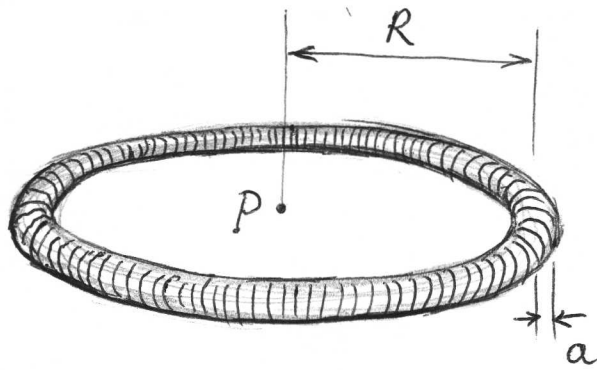
$$2\pi s E = \alpha B_0 e^{-\alpha t} \pi a^2$$

③

$$E = \frac{\alpha B_0 e^{-\alpha t} a^2}{2s}$$

$$\vec{E}(s, t) = \frac{\alpha B_0 e^{-\alpha t} a^2}{2s} \hat{\phi}$$

Pop Quiz



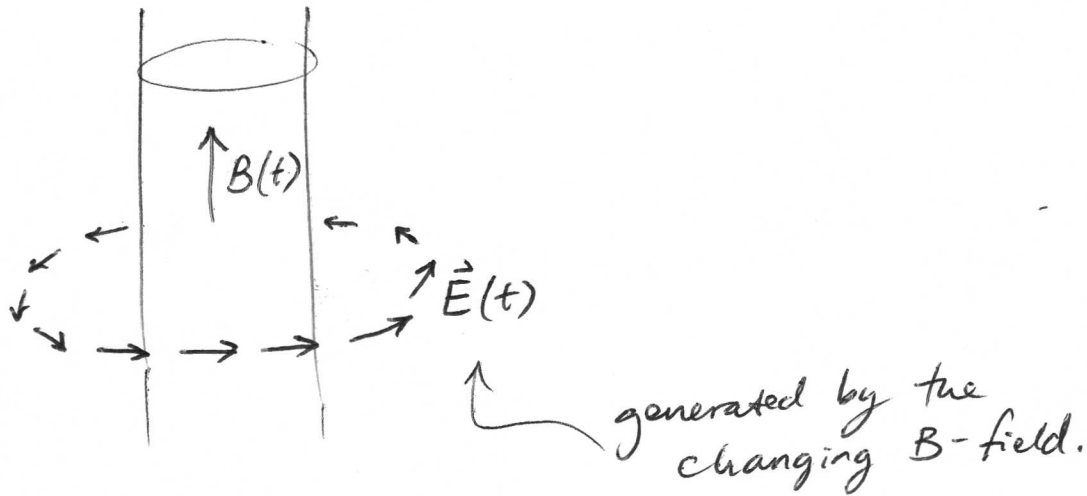
A toroidal solenoid with n turns per unit length has a small radius a and a large radius $R \gg a$.

The current in the solenoid varies with time, $I = I_0 e^{-\alpha t}$.

Find $\vec{E}(t)$ in the center of the solenoid.

(4)

Return now to the example problem

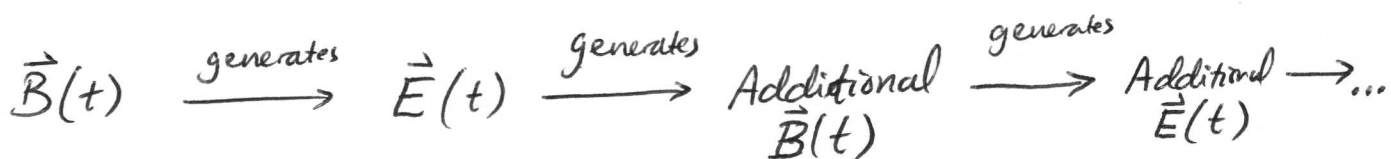


The changing \vec{E} -field will also "generate" something.

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

This term is a source of \vec{B} -field, on equal footing with \vec{J} .

To summarize



where will the calculation stop?!

In practice, we seldom go beyond $\vec{B}(t) \xrightarrow{\text{generates}} \vec{E}(t)$.
Additional corrections to \vec{B} & \vec{E} are typically tiny.