Instructor
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ELECTRODYNAMICS

Up to now our differential eqns have been

$$\nabla \cdot \vec{B} = 0$$
$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$
$$\nabla \times \vec{B} = \mu_0 \vec{J}$$
$$\nabla \times \vec{E} = 0$$

Describes Magnetostatics & Electrostatics.

We are only missing two time dependent terms to complete the theory and unify electricity & magnetism

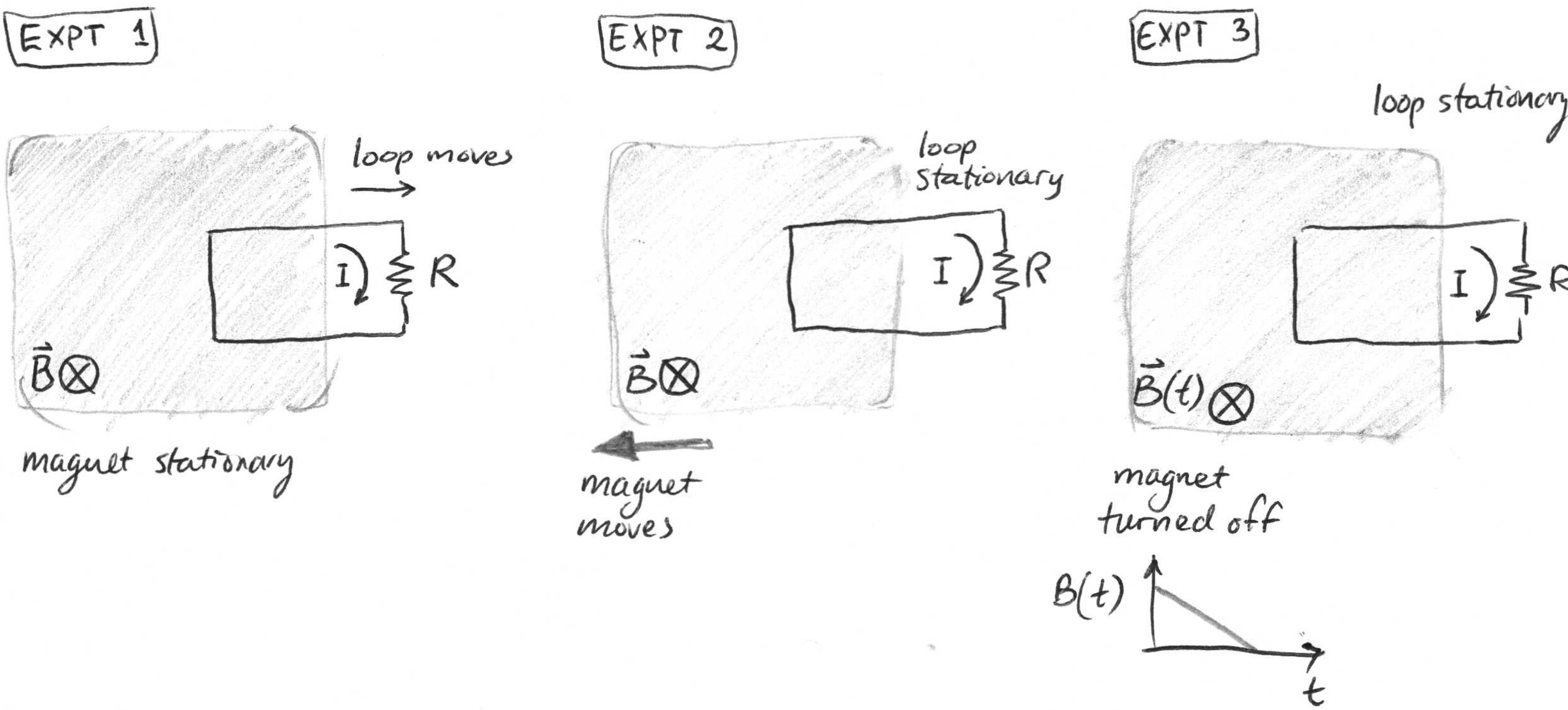
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Describes everything

"A beautifully complete and successful theory that has become a paradigm for physicists: an ideal model that other theories emulate."
— Griffith page xv.

First discuss $\frac{\partial \vec{B}}{\partial t}$.

In 1831 Faraday tried 3 expts in which he made a current "I" flow around a loop of wire.



In all three cases, Faraday found this rule to be empirically true

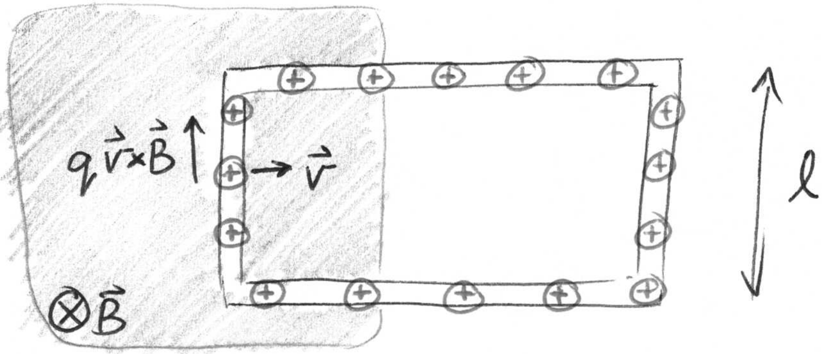
$$I = \frac{1}{R}\frac{d\Phi}{dt} \qquad \text{where } \Phi = \int_{\substack{\text{inside} \\ \text{the loop}}} \vec{B}\cdot d\vec{a}$$

It was as if $\frac{d\Phi}{dt}$ was a voltage from a battery

(Ohm's Law $I = \frac{1}{R}V$)

Today we can explain these observations using Lorentz force and/or $\nabla\times\vec{E} = -\partial\vec{B}/\partial t$.

Explaining EXPT 1



In the left arm of the loop, there is an upward force per unit charge of vB.

(analogous to an $\vec{E}$-field, but not an $\vec{E}$-field)

To calculate the analogy to voltage $\oint_{\text{around loop}} (\vec{v}\times\vec{B})\cdot d\vec{\ell}$

$$= vB\ell$$

The line integral of force per unit charge is called EMF (electromotive force). Terrible name because it is not a force!

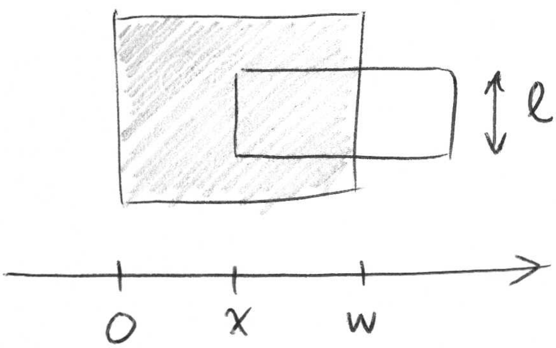
Faraday's observation that $I = \frac{1}{R}\frac{d\Phi}{dt}$ is typically summarized as

$$\boxed{\mathcal{E}MF = -\frac{d\Phi}{dt}}$$

"Faraday's Law"

For experiment 1, show that Faraday's law predicts

$$\mathcal{E}MF = vBl$$



$$\Phi = Bl(w-x)$$
$$= Blw - Blx$$

$$\frac{d\Phi}{dt} = -Bl\frac{dx}{dt} = -Blv$$

$$\Rightarrow \mathcal{E}MF = vBl$$

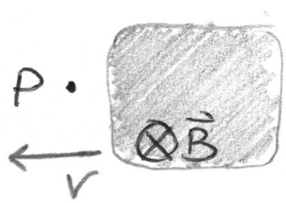
Explaining **EXPT 2**

In this expt, the wire is stationary

so $q\vec{v}\times\vec{B} = 0$.

Cannot explain the EMF using the Lorentz force.

Notice that in expt. 2, there are some points of space where $\vec{B}$ is changing



at point P