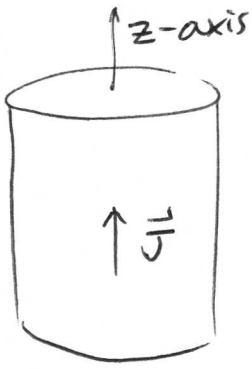
PH 632 DAY 10 Instructor Ethan Minot

Last time we derived $\nabla \times \vec{H} = \vec{J}_{free}$ where $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$



Inside a current carrying wire

$$\vec{H} = \frac{Jr}{2}\hat{\phi}$$

Now I want to know $\vec{M}$ & $\vec{B}$.

Since the metal is a linear diamagnetic material I know that $\vec{H}$, $\vec{B}$ and $\vec{M}$ all point in the same direction (either parallel or antiparallel)

$$\vec{M} = (\text{const})\vec{B}$$

↖ what shall we call this constant?

$\frac{1}{\mu_0}\frac{\chi_m}{1+\chi_m}$ seems like a good idea !?!

Note: χ_m is called magnetic susceptibility. χ_m is dimensionless, just like χ_e.

First find $\vec{M}$

$$H = \frac{1}{\mu_0} B - M$$

$$= \frac{1}{\mu_0} \frac{\mu_0(1+\chi_m)M}{\chi_m} - M$$

$$= \frac{(1+\chi_m)M - \chi_m M}{\chi_m}$$

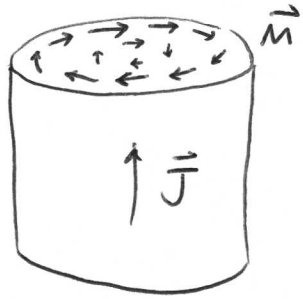
$$= \frac{M}{\chi_m}$$

$$\boxed{\vec{M} = \chi_m \vec{H}}$$ True for all linear materials.

For the current carrying wire example, $\chi_m \approx -10^{-5}$

$$\vec{M} = \frac{\chi_m J r}{2} \hat{\phi}$$

inside the wire.



Now find $\vec{B}$

$$H = \frac{1}{\mu_0} B - M$$

$$= \frac{1}{\mu_0} B - \frac{1}{\mu_0} \frac{\chi_m}{1+\chi_m} B$$

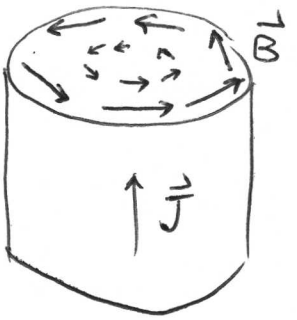
$$\mu_0 H = \frac{(1+\chi_m)B - \chi_m B}{1+\chi_m} = \frac{B}{1+\chi_m}$$

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H}$$

True for all linear materials

For the current carrying wire example, $\chi_m \approx -10^{-5}$

$$\vec{B} = \mu_0 (1 + \chi_m) \frac{Jr}{2} \hat{\phi}$$



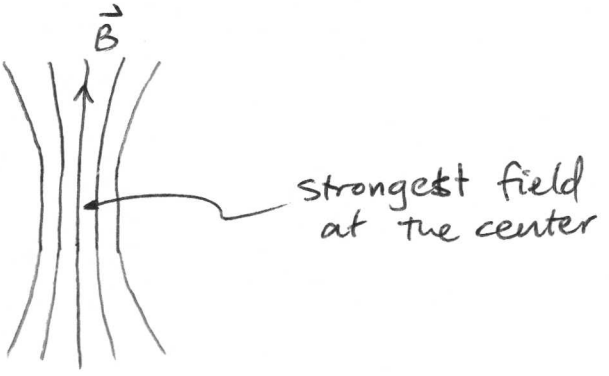
Note: $\vec{B}$ is opposite direction as $\vec{M}$.

$\vec{B}$ inside the wire is slightly less (1 part in 10^5) than you would guess if you didn't know about diamagnetism.

Pop Quiz

Pop Quiz Day 10

A frog is placed near the center of a 20 Tesla magnet



Frogs, like people, are mostly water. $\chi_m \approx -10^{-5}$

a) Estimate the net dipole moment of the frogs $\vec{M}V$, where $\vec{M}$ is magnetization and V is volume.

b) Is the frog attracted or repelled from the region of highest field?

c) Consider a material with $\chi_m = -1$. How would it behave compared to the frog?