PH631, Fall 2015

HOMEWORK SET#6

due Friday Nov 13, 2015

① a) The charge distribution in Fig. 1 has a dipole moment $\vec{p} = q\vec{d}$

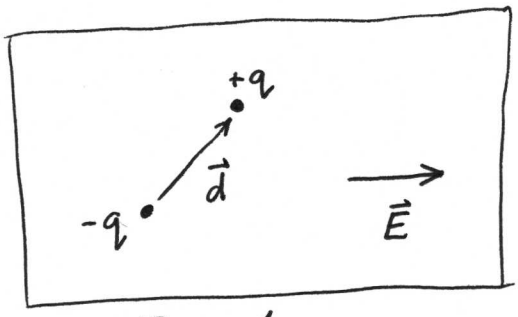


Figure 1

Show that the torque on the dipole is $\vec{\tau} = \vec{p} \times \vec{E}$

b) In Fig. 2, $\vec{p}_1$ and $\vec{p}_2$ are (perfect) dipoles a distance r apart. What is the torque on $\vec{p}_1$ due to $\vec{p}_2$? What is the torque on $\vec{p}_2$ due to $\vec{p}_1$?

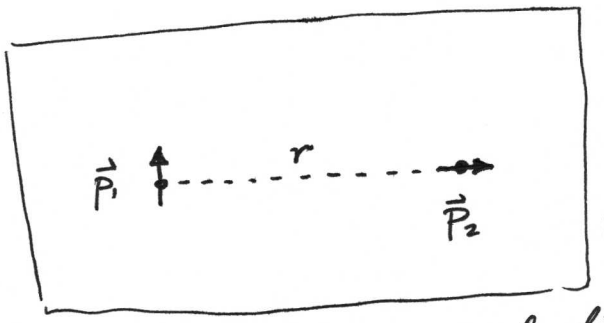


Figure 2

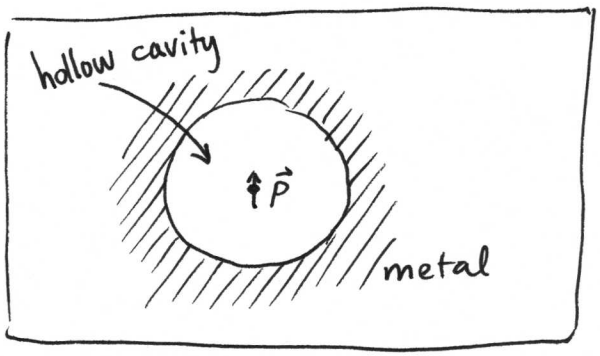
c) Show that the energy of an ideal dipole $\vec{p}$ in an electric field $\vec{E}$ is given by

$$U = -\vec{p} \cdot \vec{E}$$

(2) A perfect dipole, aligned to the z-axis, generates a potential

$$\Phi(r,\theta) = \frac{p}{4\pi\epsilon_0}\frac{1}{r^2}\cos\theta$$

This dipole is placed at the center of a grounded, metal spherical cavity, as shown in Fig 3



The radius of the hollow cavity is b. A surface charge $\sigma(b,\theta)$ is induced such that $\Phi(b,\theta) = 0$.

a) Find the potential $\Phi(r,\theta)$ for $r < b$.

Hint: $\nabla^2\left(r^l P_l(\cos\theta)\right) = 0$

b) Find $\sigma(b,\theta)$