PH631 Fall 2015

Electromagnetic Theory I

Problem Set #1

Due: 10-2-2015

1. Consider four different charged spheres of radius *a* and total charge *Q*. One is conducting, one has a uniform charge density within its volume, one has a spherically symmetric charge density that varies radially as r^2, and one has a spherically symmetric charge density that varies radially as $1/r^2$. Use Gauss's theorem to obtain the electric fields both inside and outside each sphere. Sketch the behavior of the fields as a function of radius for the first four spheres.

2. The time-averaged potential of a neutral hydrogen atom is given by

$$\Phi = \frac{q}{4\pi\varepsilon_0}\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right)$$

where *q* is the magnitude of the electronic charge, and α determines the size of the atom. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

3. A simple capacitor is a device formed by two insulated conductors adjacent to each other. If equal and opposite charges are placed on the conductors, there will be a certain difference of potential between them .The ratio of the magnitude of the charge on one conductor to the magnitude of the potential difference is called the capacitance. Using Gauss's law, calculate the capacitance of

(a) two large, flat, conducting sheets of area *A*, separated by a small distance *d*;
(b) two concentric conducting spheres with radii *a*, *b* (*b* > *a*);
(c) two concentric conducting cylinders of length *L*, large compared to their radii *a*, *b* (*b* > *a*);
(d) What is the inner diameter of the outer conductor in an air-filled coaxial cable whose center conductor is a cylindrical wire of diameter 1 mm and whose capacitance is 3×10^{-11} F/m? What if 3×10^{-12} F/m?

4. Prove the mean value theorem: For charge-free space the value of the electrostatic potential at any point is equal to the average of the potential over the surface of any sphere centered on that point.