DAY 9

PH 631
2015

Instructor:
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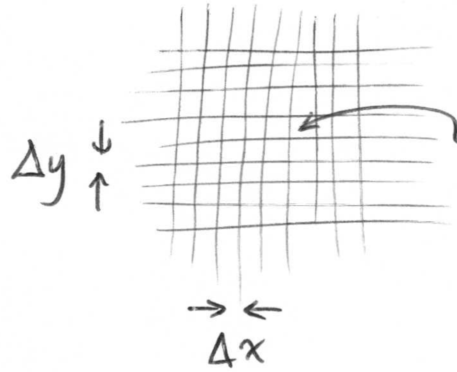
## NUMERICAL RELAXATION METHOD




pixel $i, j$
has potential $\Phi_{ij} = \Phi(x, y)$

we require $\nabla^2 \Phi = 0$

$$\Rightarrow \quad \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$

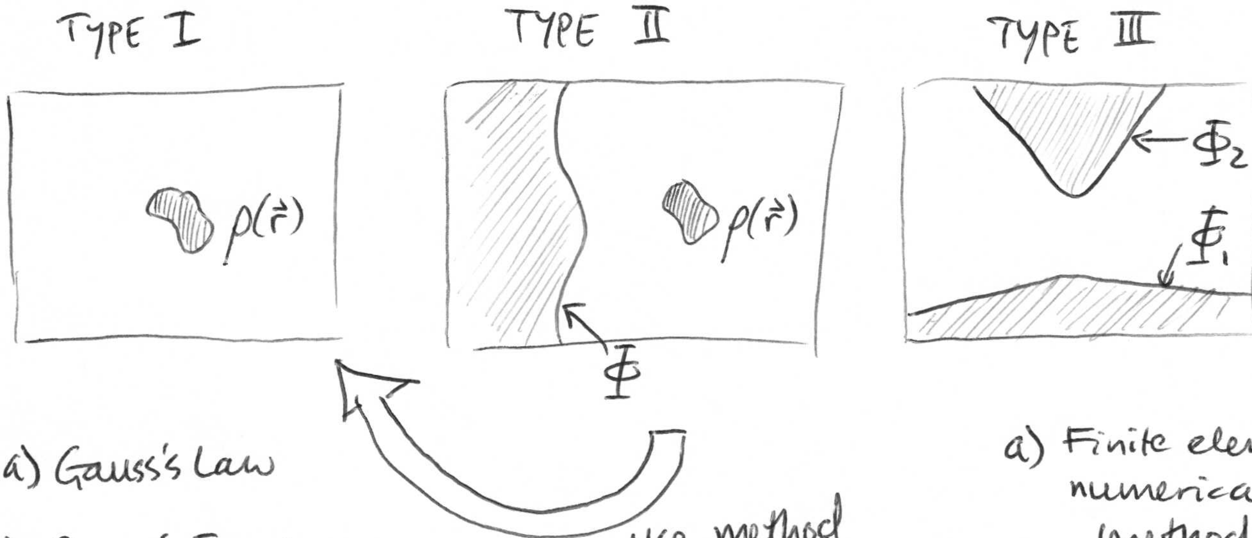
$$\Rightarrow \quad \frac{\dfrac{(\Phi_{i+1,j} - \Phi_{ij})}{\Delta x} - \dfrac{(\Phi_{ij} - \Phi_{i-1,j})}{\Delta x}}{\Delta x} + \frac{\dfrac{(\Phi_{ij+1} - \Phi_{ij})}{\Delta y} - \dfrac{(\Phi_{ij} - \Phi_{ij-1})}{\Delta y}}{\Delta y} = 0$$

If we set $\Delta x = \Delta y$ (convenient choice for numerical methods)

then $-4\Phi_{ij} + \Phi_{i+1\,j} + \Phi_{i-1\,j} + \Phi_{i\,j+1} + \Phi_{i\,j-1} = 0$

$$\boxed{\Phi_{ij} = \frac{\Phi_{i+1\,j} + \Phi_{i-1\,j} + \Phi_{i\,j+1} + \Phi_{i\,j-1}}{4}}$$

Where are we in the big picture of this class?




a) Gauss's Law

b) Green's Function

(Remember to make use of symmetry arguments and superposition principle)

c) Multipole expansion

(to estimate $\Phi(\vec{r})$ far away from a charge distribution)

Use method of Images to convert to type I

a) Finite element numerical method

b) Summation of orthogonal functions.

__Next__: Summation of orthogonal functions.

# SUMMATION OF ORTHOGONAL FUNCTIONS

we know $\nabla^2 \Phi = 0$ in VOI if the volume is empty.

Find a convenient & complete set of orthogonal functions
$f_1(x,y,z),\ f_2(x,y,z),\ f_3(x,y,z),\ \ldots$

that satisfy
$$\nabla^2 f_1 = 0$$
$$\nabla^2 f_2 = 0$$
$$\nabla^2 f_3 = 0$$
$$\vdots$$

Now $\Phi(x,y,z) = A_1 f_1(x,y,z) + A_2 f_2(x,y,z) + A_3 f_3(x,y,z) + \ldots$

where $A_1, A_2, A_3 \ldots$ are constants that are chosen to match the B.C.s on $\Phi(x,y,z)$

$\boxed{\text{CARTESIAN COORDINATES}}$ Convenient set of functions


$\sin \alpha x \, e^{-\alpha y}$

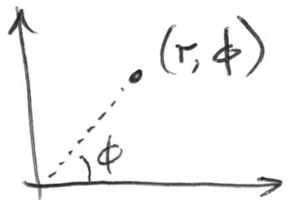
$\sin \beta y \, e^{-\beta x}$

$\cos \gamma x \, e^{-\gamma y}$

$\cos \xi y \, e^{-\xi x}$

These functions form a complete set.

# POLAR COORDINATES




Convenient set of functions

$r^\nu \sin \nu\phi$

$r^\nu \cos \nu\phi$

($\nu$ can be positive or negative)

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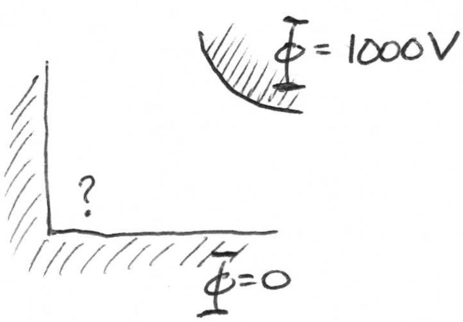
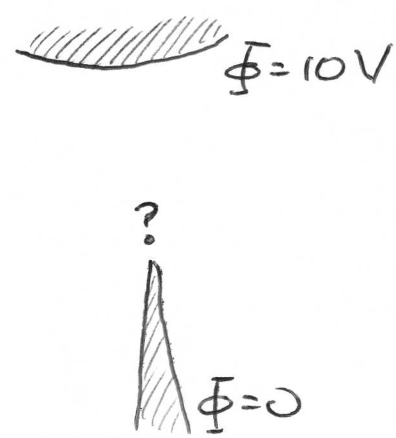
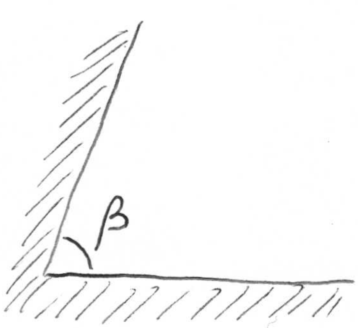
Example 1 What is the potential near an insider corner or an outside corner?




Answer: Consider an arbitrary corner described by ~~with~~ an angle $\beta$. Put origin at corner.




The volume of interest is $0 < \phi < \beta$.

$\Phi(r,\phi)$ can be expressed by the summation

$$\Phi(r,\phi) = \sum_\nu A_\nu r^\nu \sin \nu\phi$$