DAY 7

PH 631

2015

Instructor:
Ethan Minot

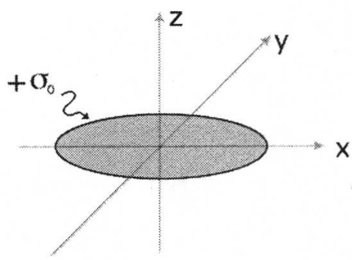
Start with pop quiz

(2)

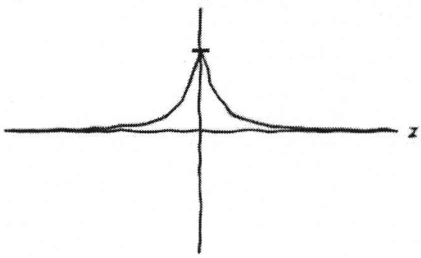
PH631 Pop Quiz #7

Name:

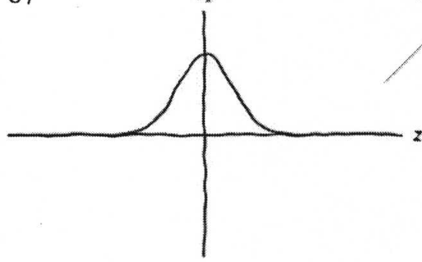
Consider a uniform flat, infinitely thin disk of radius *R* carrying a uniform positive surface charge density $+\sigma_0$ as in the figure. Which of the following qualitative graphs best represent the relative sign and magnitude of E_z as you move away from the disk along the z axis.



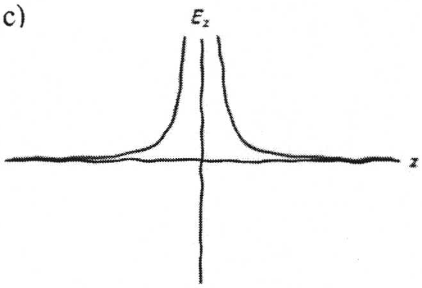
a)



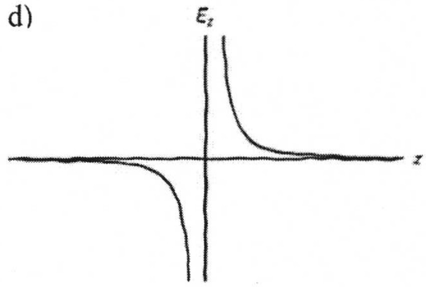
b)



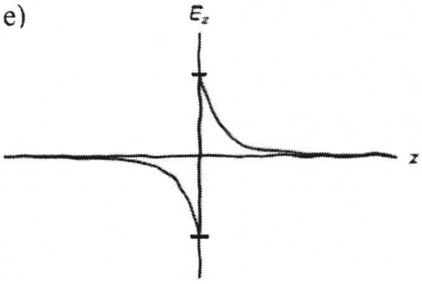
c)



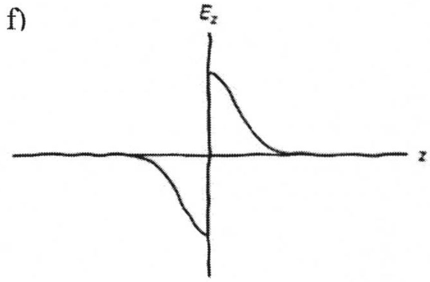
d)



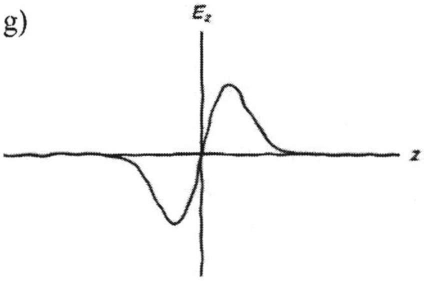
e)



f)

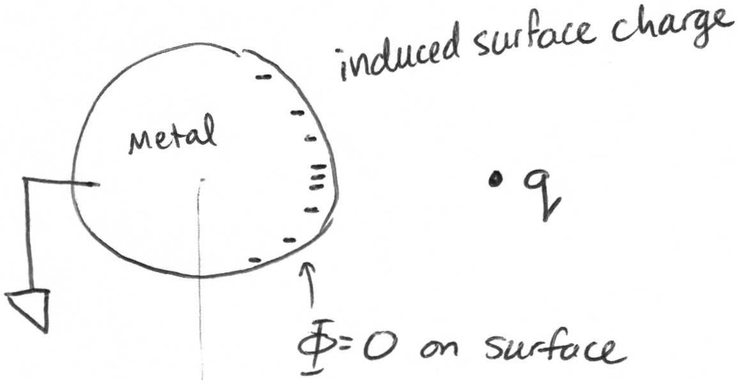


g)



h) None of these
(please explain in the space below)

ANOTHER GEOMETRY FOR METHOD OF IMAGES



VOI is ~~outs~~ the space around the metal sphere

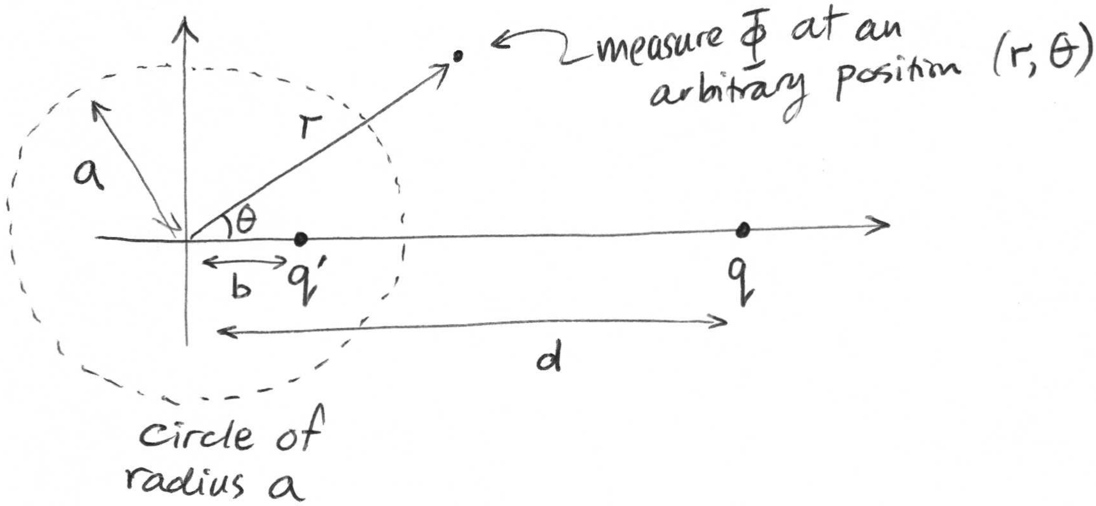
Map onto an "easier" charge distribution

By choosing the correct magnitude of q', and placing it off-center from the metal sphere center, I can create a spherical surface of $\Phi = 0$ that matches the boundary condition of the VOI.

- See computer simulation on the web, PhET website.

Quantitative analysis:

Find q' and its position relative to the radius of the sphere and the magnitude/position of q.



$$\Phi(r,\theta) = \frac{1}{4\pi\epsilon_0}\left(\frac{q'}{\sqrt{r^2\sin^2\theta + (r\cos\theta - b)^2}} + \frac{q}{\sqrt{r^2\sin^2\theta + (r\cos\theta - d)^2}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q'}{\sqrt{r^2 + b^2 - 2rb\cos\theta}} + \frac{q}{\sqrt{r^2 + d^2 - 2rd\cos\theta}}\right)$$

Set the radius $r = a$ such that $\Phi(a, \theta) = 0$

i.e. $$\frac{q'}{\sqrt{a^2+b^2-2ab\cos\theta}} + \frac{q}{\sqrt{a^2+d^2-2ad\cos\theta}} = 0$$

$$\Rightarrow \sqrt{\frac{a^2+d^2-2ad\cos\theta}{a^2+b^2-2ab\cos\theta}} = -\frac{q}{q'} = \text{constant}$$

At what value of b (position of image charge) will LHS be constant?

i.e. We must choose b so that

$$LHS = \sqrt{\frac{\alpha f(\cos\theta)}{f(\cos\theta)}}$$ where α is a const and $f(\cos\theta)$ is a function of $\cos\theta$.

$$\sqrt{\frac{d^2\left(1+\frac{a^2}{d^2}-\frac{2a}{d}\cos\theta\right)}{a^2\left(1+\frac{b^2}{a^2}-\frac{2b}{a}\cos\theta\right)}} = -\frac{q}{q'}$$

set $\frac{a}{d} = \frac{b}{a}$, $\boxed{b = \frac{a^2}{d}}$

then $\sqrt{\frac{d^2}{a^2}} = -\frac{q}{q'}$ $\boxed{q' = -\frac{a}{d}q}$