

DAY 7

①

PH 631
2015

Instructor:
Ethan Minot

Start with pop quiz

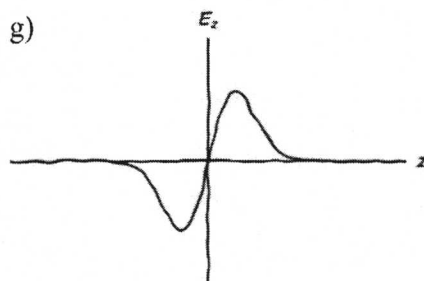
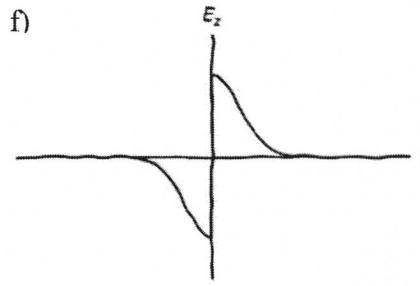
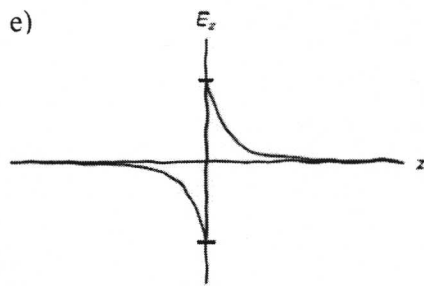
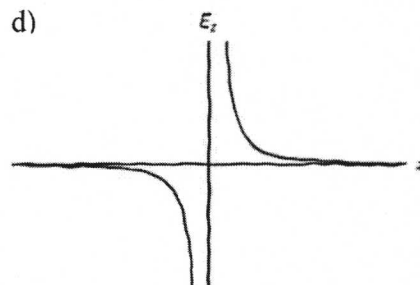
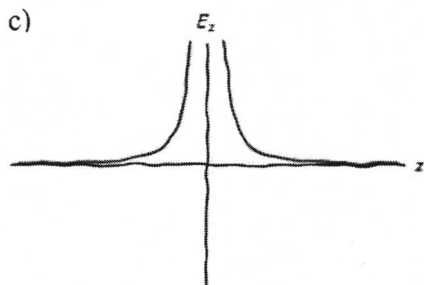
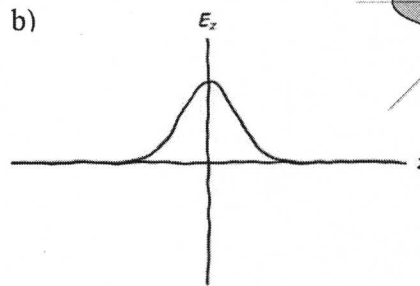
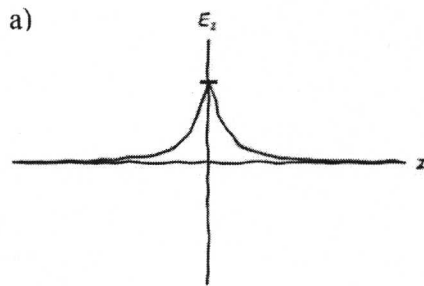
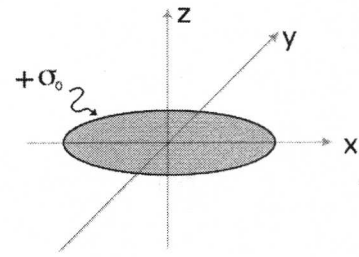
2

PH631

Pop Quiz #7

Name:

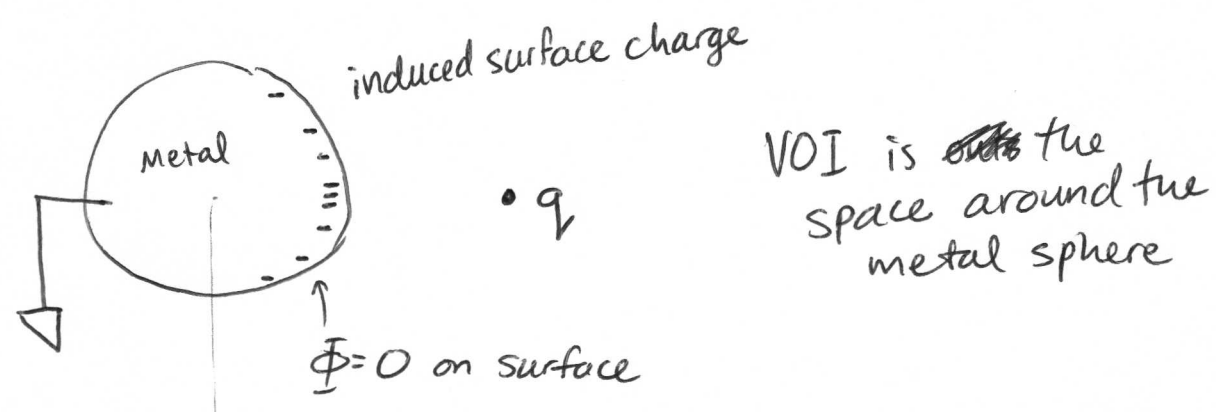
Consider a uniform flat, infinitely thin disk of radius R carrying a uniform positive surface charge density $+\sigma_0$ as in the figure. Which of the following qualitative graphs best represent the relative sign and magnitude of E_z as you move away from the disk along the z axis.



h) None of these
(please explain in the space below)

3

ANOTHER GEOMETRY FOR METHOD OF IMAGES



Map onto an "easier" charge distribution



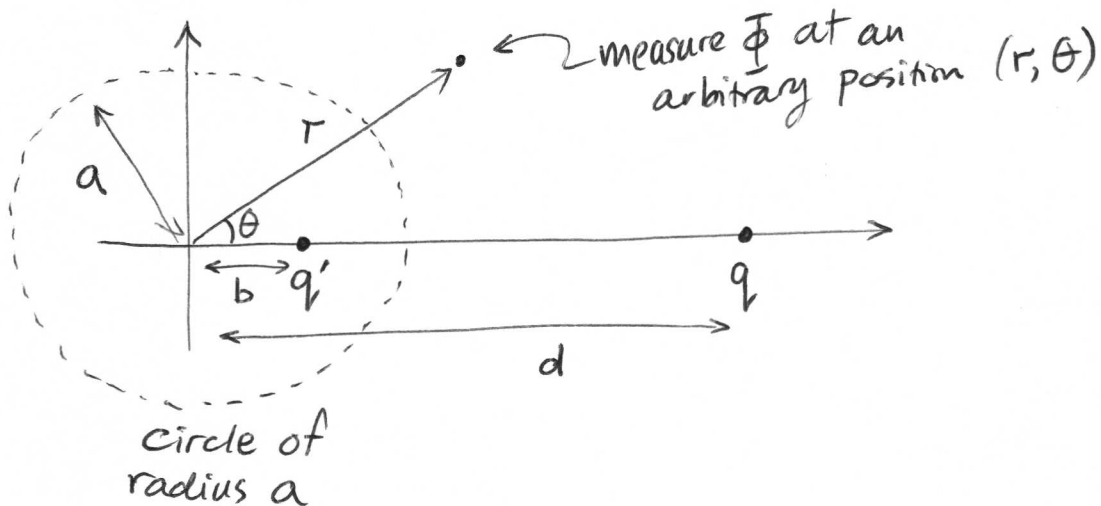
By choosing the correct magnitude of q' , and placing it off-center from the metal sphere center, I can create a spherical surface of $\Phi = 0$ that matches the boundary condition of the VOI.

- See computer simulation on the web, PhET website.

(4)

Quantitative analysis:

Find q' and its position relative to the radius of the sphere and the ^{magnitude/} position of q .



$$\Phi(r, \theta) = \frac{1}{4\pi\epsilon_0} \left(\frac{q'}{\sqrt{r^2 \sin^2 \theta + (r \cos \theta - b)^2}} + \frac{q}{\sqrt{r^2 \sin^2 \theta + (r \cos \theta - d)^2}} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{q'}{\sqrt{r^2 + b^2 - 2rb \cos \theta}} + \frac{q}{\sqrt{r^2 + d^2 - 2rd \cos \theta}} \right)$$

Set the radius $r = a$ such that $\Phi(a, \theta) = 0$

i.e.
$$\frac{q'}{\sqrt{a^2 + b^2 - 2ab \cos \theta}} + \frac{q}{\sqrt{a^2 + d^2 - 2ad \cos \theta}} = 0$$

(5)

$$\Rightarrow \sqrt{\frac{a^2 + d^2 - 2ad \cos \theta}{a^2 + b^2 - 2ab \cos \theta}} = -\frac{q}{q'} = \text{constant}$$

At what value of b (position of image charge) will LHS be constant?

i.e. We must choose b so that

$$\text{LHS} = \sqrt{\frac{\alpha f(\cos \theta)}{f(\cos \theta)}} \quad \text{where } \alpha \text{ is a const and } f(\cos \theta) \text{ is a function of } \cos \theta.$$

$$\sqrt{\frac{d^2 \left(1 + \frac{a^2}{d^2} - \frac{2a}{d} \cos \theta \right)}{a^2 \left(1 + \frac{b^2}{a^2} - \frac{2b}{a} \cos \theta \right)}} = -\frac{q}{q'}$$

set $\frac{a}{d} = \frac{b}{a}$, $\boxed{b = \frac{a^2}{d}}$

then $\sqrt{\frac{d^2}{a^2}} = -\frac{q}{q'}$ $\boxed{q' = -\frac{a}{d} q}$