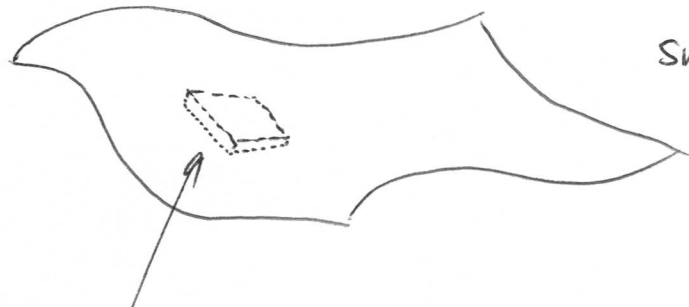


DAY 6

PH 631  
2015

Instructor  
Stuart Minot

# $\vec{E}$ -FIELD NEAR SURFACE CHARGE

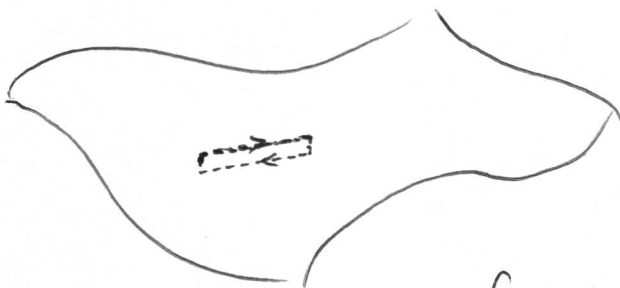


smoothly varying surface charge  $\sigma$   
Surface can be curved.

Water-thin Gaussian Pill-box surface  
(used for Gauss's Law). Top surface is above  
Bottom surface is below.

$$\int_{\text{Pill box Surf}} \vec{E} \cdot d\vec{a} = (E_{\perp \text{ above}} - E_{\perp \text{ below}})A = \frac{\sigma A}{\epsilon_0}$$

$$E_{\perp \text{ above}} - E_{\perp \text{ below}} = \frac{\sigma}{\epsilon_0} \quad \text{--- ①}$$



Consider a <sup>closed</sup> path integral  
that goes just above and  
just below the same surface

$$\oint \vec{E} \cdot d\vec{\ell} = 0 \quad \text{because} \quad \vec{E} = -\nabla\Phi$$

$$\Rightarrow \boxed{E_{\parallel \text{ above}} = E_{\parallel \text{ below}}} \quad \text{---} \quad \textcircled{2}$$

Combining ① & ②

$$\boxed{\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\epsilon_0} \hat{n}}$$

$\hat{n}$  is normal to surface.

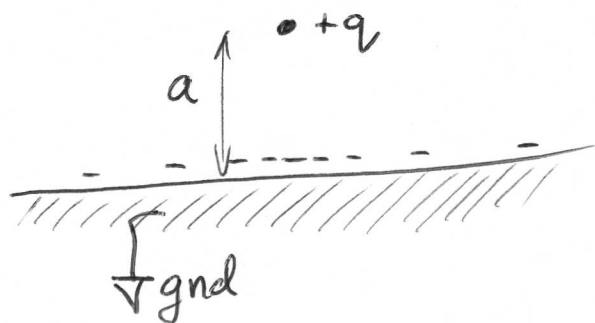
Special case when  $\sigma$  is on the surface of a metal

$$\boxed{\vec{E}_{\text{above metal}} = \frac{\sigma}{\epsilon_0} \hat{n}}$$

TRUE WHENEVER  $\vec{E}$  IS MEASURED VERY CLOSE TO A METAL SURFACE

### Application of these results

Calculate  $\sigma(x,y)$  on metal under a point charge.



Step 1: Find  $\Phi(x,y,z)$  in VOI

Step 2: Find  $\vec{E}_{\text{above}}$  at the metal surface

Step 3: Use  $\vec{E}_{\text{above}} = \frac{\sigma}{\epsilon_0} \hat{n}$

$$\Phi(x, y, z) = \frac{q}{4\pi\epsilon_0} \left( \frac{1}{\sqrt{x^2 + y^2 + (z-a)^2}} - \frac{1}{\sqrt{x^2 + y^2 + (z+a)^2}} \right)$$

$$\vec{E} = -\nabla\Phi$$

$$= -\frac{\partial\Phi}{\partial x} \hat{x} - \frac{\partial\Phi}{\partial y} \hat{y} - \frac{\partial\Phi}{\partial z} \hat{z}$$

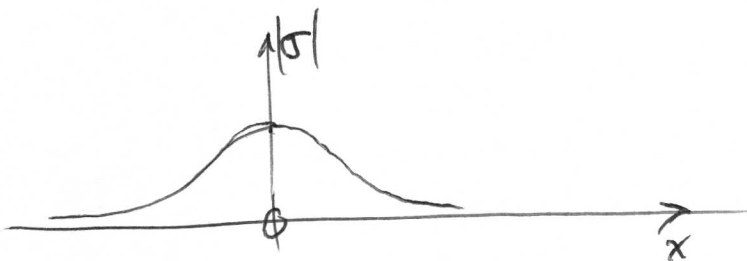
= 0 at surface

$$E_z \Big|_{z=0} = -\frac{\partial\Phi}{\partial z} \Big|_{z=0}$$

$$= -\frac{q}{4\pi\epsilon_0} \left( \frac{-\frac{1}{2} \cdot 2(z-a)}{(x^2 + y^2 + (z-a)^2)^{3/2}} - \frac{-\frac{1}{2} \cdot 2(z+a)}{(x^2 + y^2 + (z+a)^2)^{3/2}} \right) \Big|_{z=0}$$

$$= \frac{-q}{4\pi\epsilon_0} \left[ \frac{2a}{(x^2 + y^2 + a^2)^{3/2}} \right]$$

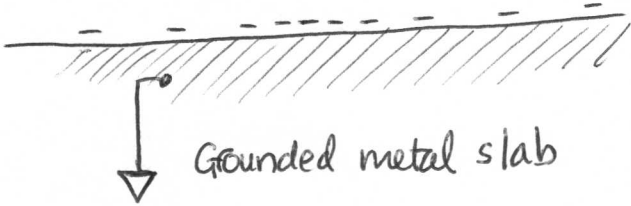
$$\sigma = \epsilon_0 E_z = \frac{-q}{4\pi} \left( \frac{2a}{(x^2 + y^2 + a^2)^{3/2}} \right)$$



DAY 6

(4)  
BLACK BOARD QUESTION

• +q



Show that the net charge on the metal surface is  $-q$ .