DAY 5

PH 631, 2015

Instructor
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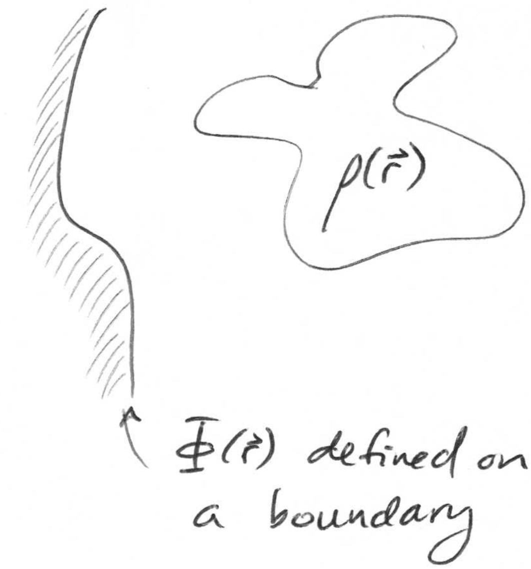
Type II problems



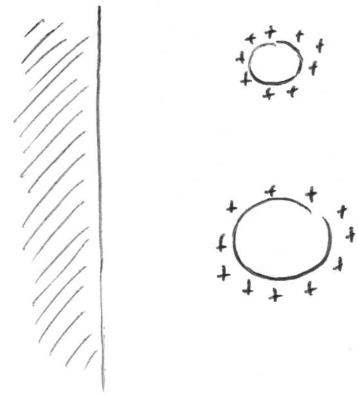
How do we apply the Poisson eqn

$$\nabla^2 \Phi(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0} \qquad ?$$

Option #1 : Map the Type II problem onto a Type I problem that is easier to solve.

i.e. "Method of images"

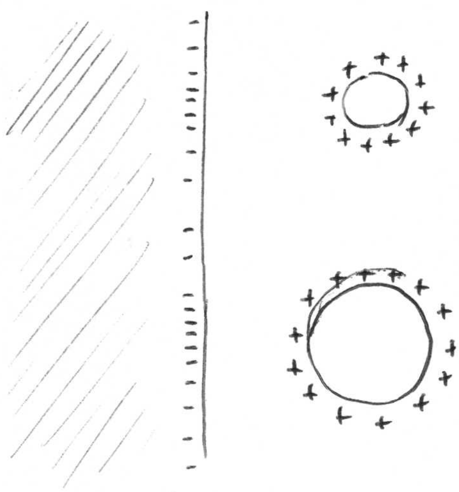
EXAMPLE



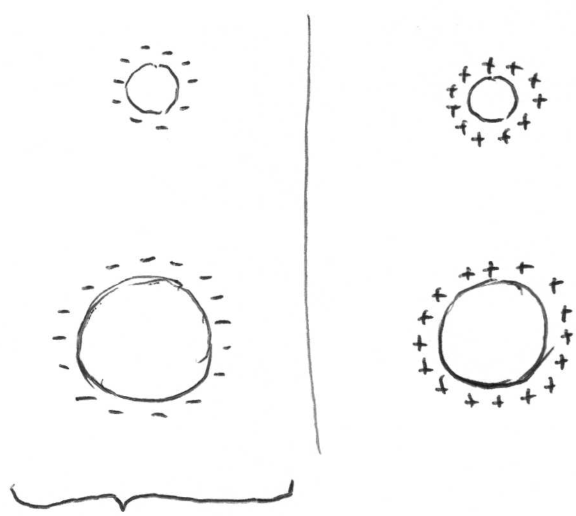
$$\Phi(\vec{r}) = \begin{cases} 0 & x<0 \\ ? & x>0, \text{ the volume of interest, VOI.} \end{cases}$$

What actually happens

A type I problem that gives the same $\Phi(\vec{r})$ in the VOI



Free electrons move to the surface of the metal, attracted by the positive spheres.



Fictional $\rho(\vec{r})$ outside VOI that generates the correct $\Phi(\vec{r})$ in the VOI.

Outside the VOI is like Las Vegas.
No one will question what you did outside the VOI
as long as $\Phi(\vec{r})$ is correct inside the VOI

UNIQUENESS THM:

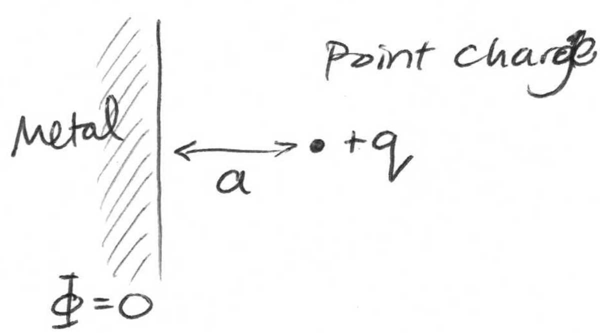
The solution to $\nabla^2 \Phi = -\frac{\rho(\vec{r})}{\epsilon_0}$ inside the VOI is unique if either

(a) Φ matches the boundary condition (BCs) on the VOI surface

(b) $\frac{\partial \Phi}{\partial n}$ matches the BCs on the VOI surface

(n is direction normal to surface).

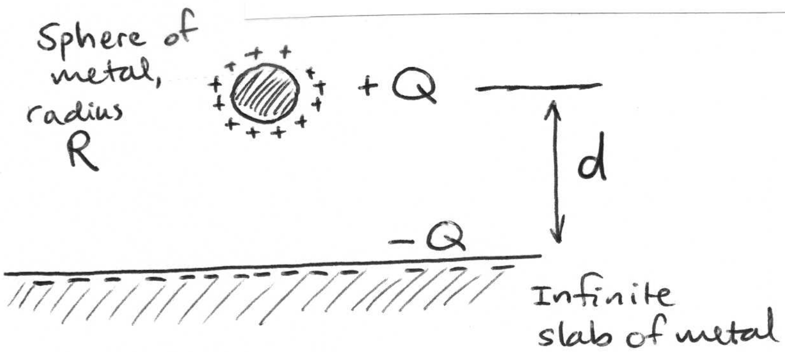
Easiest example



$$\Phi(\vec{r}) = \begin{cases} 0 & x<0 \\ \dfrac{q}{4\pi\epsilon_0 |\vec{r}-a\hat{x}|} - \underbrace{\dfrac{q}{4\pi\epsilon_0 |\vec{r}+a\hat{x}|}} & x>0, \text{ the VOI} \end{cases}$$

This term comes from a fictional charge outside the VOI.

BLACKBOARD (4) PROBLEM



When $d \gg R$, the charge $+Q$ distributes uniformly on the sphere. Calculate the capacitance between the two pieces of metal.