DAY 4

2015

PH 631

Instructor:
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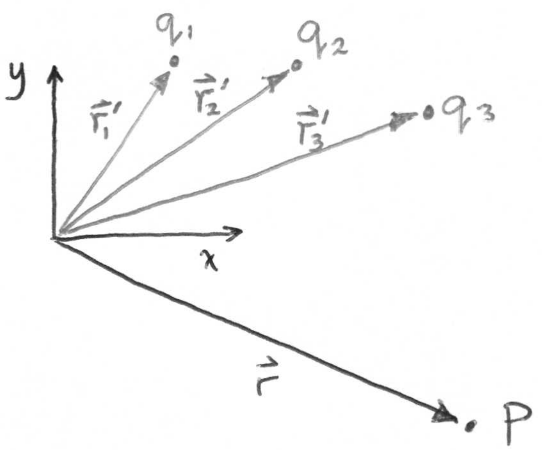
You are ~~given~~ told $\rho(\vec{r})$ everywhere and asked to find $\Phi(\vec{r})$.

If Gauss's Law doesn't help try

$$\boxed{\Phi(\vec{r}) = \int_{\text{every where}} \frac{\rho(\vec{r}\,')\, d^3\vec{r}\,'}{4\pi\epsilon_0 |\vec{r}-\vec{r}\,'|}} \qquad \text{—— } ①$$

To derive ①, start with a discrete charge distribution

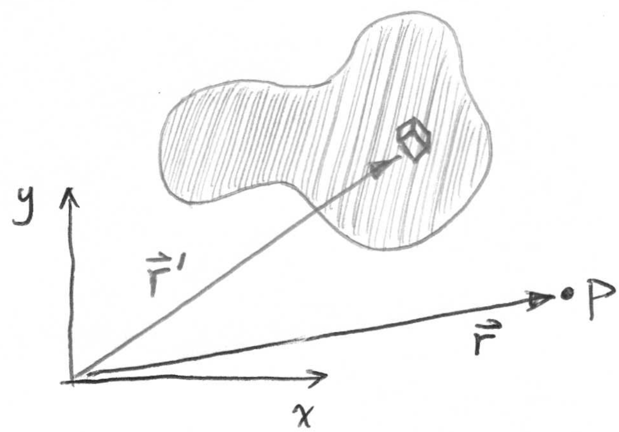


The primed vectors tell us the location of charge 1, 2 and 3.

The "not primed" vector tells us the location of point P. Find Φ at point P.

$$\Phi(\vec{r}) = \frac{q_1}{4\pi\epsilon_0 |\vec{r}-\vec{r}_1\,'|} + \frac{q_2}{4\pi\epsilon_0 |\vec{r}-\vec{r}_2\,'|} + \frac{q_3}{4\pi\epsilon_0 |\vec{r}-\vec{r}_3\,'|}$$

Now take the charges and spread them out into a continuous distribution $\rho(\vec{r}\,')$



A small volume $d^3\vec{r}\,'$ contains charge $\rho(\vec{r}\,')\,d^3\vec{r}\,'$

The charge in volume $d^3\vec{r}\,'$ ~~creates a~~ generates a potential

$$\frac{\rho(\vec{r}\,')d^3\vec{r}\,'}{4\pi\epsilon_0|\vec{r}-\vec{r}\,'|} \text{ at point P.}$$

Add together the potential generated by every piece of the charge distribution

$$\Phi(\vec{r}) = \int_{\text{everywhere}} \frac{\rho(\vec{r}\,')\,d^3\vec{r}\,'}{4\pi\epsilon_0|\vec{r}-\vec{r}\,'|}$$

This is the concept of "chop and add".

A fancier name is "Green's function integral"

If the charge distribution is limited to a 2d surface

$$\Phi(\vec{r}) = \int_{\text{surface}} \frac{\sigma(\vec{r}\,')\,da'}{4\pi\epsilon_0 |\vec{r} - \vec{r}\,'|}$$

If the charge distribution is restricted to a line

$$\Phi(\vec{r}) = \int_{\text{line}} \frac{\lambda(\vec{r}\,')\,dl'}{4\pi\epsilon_0 |\vec{r} - \vec{r}\,'|}$$

Intro to Green's Functions

$$\Phi(\vec{r}) = \int G(\vec{r},\vec{r}\,')\rho(\vec{r}\,')d^3\vec{r}\,'$$

$$\begin{tikzcd} \rho(\vec{r}) \arrow[r, bend left] & \Phi(\vec{r}) \arrow[l, bend left] \end{tikzcd}$$

Poisson Eq^n

$$\rho(\vec{r}) = -\epsilon_0 \nabla^2 \Phi(\vec{r})$$

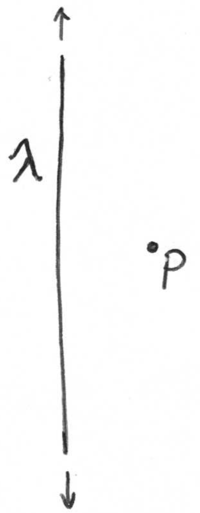
In one direction, the transformation is done by differentiating.

In the other direction, the transformation is done by integrating.

Many differential eqns in Physics (including the Poisson eq^n, the diffusion eq^n & the Schroedinger eq^n) have an associated Greens function $G(\vec{r},\vec{r}\,')$

POP QUIZ PH 631 DAY 4

Name ______________________



An infinitely long line of charge is uniformly distributed such that each segment of length L has charge λL.

Use a Green's function integral to find the potential at point P.