

DAY 4

2015

PH 631

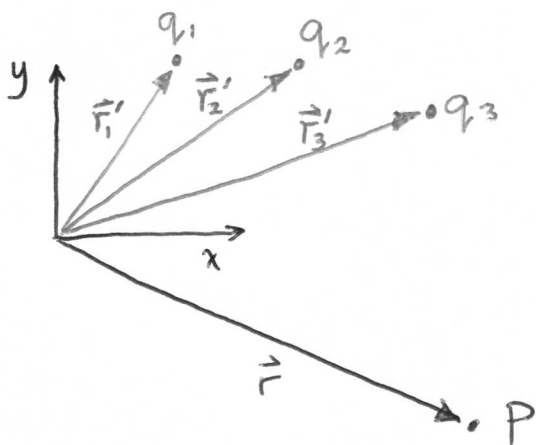
Instructor:
Ethan Minot

You are ~~given~~^{told} $\rho(\vec{r})$ everywhere
and asked to find $\Phi(\vec{r})$.

If Gauss's Law doesn't help try

$$\Phi(\vec{r}) = \int_{\text{everywhere}} \frac{\rho(\vec{r}') d^3\vec{r}'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|} \quad \text{--- ①}$$

To derive ①, start with a discrete charge distribution



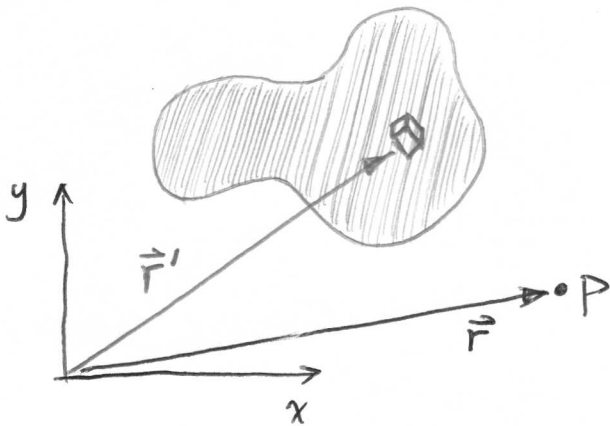
The primed vectors tell us the location of charge 1, 2 and 3.

The "not primed" vector tells us the location of point P. Find Φ at point P.

$$\Phi(\vec{r}) = \frac{q_1}{4\pi\epsilon_0 |\vec{r} - \vec{r}'_1|} + \frac{q_2}{4\pi\epsilon_0 |\vec{r} - \vec{r}'_2|} + \frac{q_3}{4\pi\epsilon_0 |\vec{r} - \vec{r}'_3|}$$

(2)

Now take the charges and spread them out into a continuous distribution $\rho(\vec{r}')$



A small volume $d^3\vec{r}'$ contains charge $\rho(\vec{r}')d^3\vec{r}'$

The charge in volume $d^3\vec{r}'$ ~~creates~~ a generates a potential

$$\frac{\rho(\vec{r}')d^3\vec{r}'}{4\pi\epsilon_0|\vec{r}-\vec{r}'|} \text{ at point P.}$$

Add together the potential generated by every piece of the charge distribution

$$\Phi(\vec{r}) = \int_{\text{every where}} \frac{\rho(\vec{r}')d^3\vec{r}'}{4\pi\epsilon_0|\vec{r}-\vec{r}'|}$$

This is the concept of "chop and add".

A fancier name is "Green's function integral"

③

If the charge distribution is limited to a 2d surface

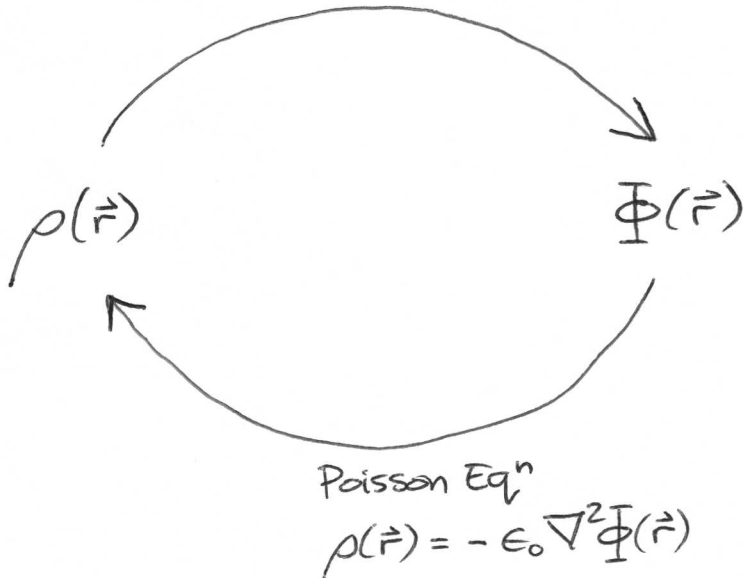
$$\Phi(\vec{r}) = \int_{\text{surface}} \frac{\sigma(\vec{r}') da'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$$

If the charge distribution is restricted to a line

$$\Phi(\vec{r}) = \int_{\text{line}} \frac{\lambda(\vec{r}') dl'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$$

INTRO TO GREEN'S FUNCTIONS

$$\Phi(\vec{r}) = \int G(\vec{r}, \vec{r}') \rho(\vec{r}') d^3\vec{r}'$$

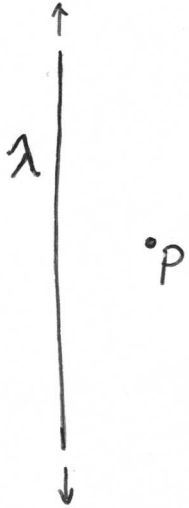


In one direction, the transformation is done by differentiating.

In the other direction, the transformation is done by integrating.

Many differential eqns in physics (including the Poisson eqn, the diffusion eqn & the Schrodinger eqn) have an associated Green's function $G(\vec{r}, \vec{r}')$

Name _____



An infinitely long line of charge is uniformly distributed such that each segment of length L has charge λL .

Use a Green's function integral to find the potential at point P .