

## MATH METHODS FOR PHYSICS

## DIMENSIONAL REASONING

- ① The argument of  $\sin$ ,  $\cos$ ,  $\exp$  etc. is dimensionless.
- ② Derivative w.r.t. length gives something per length.
- ③ Dimensions of LHS = Dimensions of RHS

Exercise: Use dimensional reasoning to catch mistakes in the following ~~one~~ statements

$$\frac{\partial}{\partial r} \frac{1}{r} = \ln(r)$$

$$\frac{\partial^2}{\partial x^2} \sin kx = k \cos kx$$

$$\frac{\partial}{\partial x} \sin kx = \frac{1}{k} \sin kx$$

POP QUIZ

(2)  
PH 631

DAY 2

Name \_\_\_\_\_

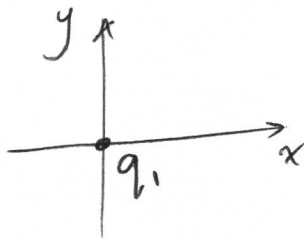
This question is closed book. Don't look at your notes.  
But do talk to your classmates.

a) Write down the Poisson Eq<sup>n</sup>

b) Derive Gauss's Law

(3)

# COULOMB POTENTIAL SATISFIES POISSON EQN



$$\Phi(\vec{r}) = \frac{q_1}{4\pi\epsilon_0|\vec{r}|}$$

$$\rho(\vec{r}) = q_1 \delta(\vec{r})$$

Show that  

$$\nabla^2 \Phi = -\frac{\rho(\vec{r})}{\epsilon_0}$$

Our task is to show

$$\nabla^2 \left( \frac{q_1}{4\pi\epsilon_0 r} \right) = -\frac{q_1 \delta(\vec{r})}{\epsilon_0}$$

or, equivalently 
$$\nabla^2 \left( \frac{1}{4\pi r} \right) = -\delta(\vec{r})$$

On day 1 we showed that  $\nabla^2 \left( \frac{1}{r} \right) = 0$  when  $r \neq 0$ .

All that remains to show is that

$$\int_{\text{vol}} \nabla^2 \left( \frac{1}{4\pi r} \right) d^3\vec{r} = -1$$

$$\text{LHS} = \int_{\text{vol}} \nabla \cdot \nabla \left( \frac{1}{4\pi r} \right) d^3\vec{r}$$

$$= \int_{\text{surf}} \nabla \left( \frac{1}{4\pi r} \right) \cdot d\vec{a}$$

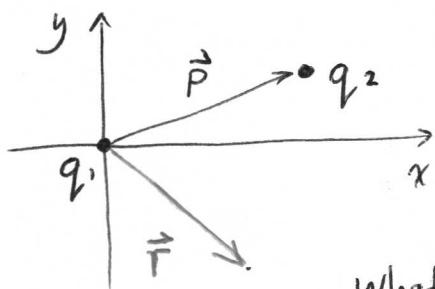
} Stokes thm allows us to avoid the discontinuity at  $r=0$ .

(4)

Choose a spherical surface of radius  $R$  centered on the origin.

$$\begin{aligned}
 &= \int_{\text{surf}} \frac{-1}{4\pi r^2} \hat{r} \cdot d\vec{a} \\
 &= \frac{-1}{4\pi R^2} 4\pi R^2 \\
 &= -1 \quad \text{Q.E.D.}
 \end{aligned}$$

## SUPER POSITION PRINCIPLE



What is  $\Phi$  at position  $\vec{r}$ ?

$$\Phi(\vec{r}) = \frac{q_1}{4\pi\epsilon_0|\vec{r}|} + \frac{q_2}{4\pi\epsilon_0|\vec{r}-\vec{p}|}$$

How do I prove this is true?

Simply check that Poisson Eqn is satisfied.

$$\nabla^2 \Phi = -\frac{q_1}{\epsilon_0} \delta(\vec{r}) + \frac{-q_2}{\epsilon_0} \delta(\vec{r}-\vec{p}) = -\frac{1}{\epsilon_0} \rho(\vec{r}) \quad \checkmark$$

⑤

The superposition principle extends to a distribution of 10 charges, 1000 charges, even a continuous distribution of charge.

We will use it again and again.