PH 631 DAY 2

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MATH METHODS FOR PHYSICS

DIMENSIONAL REASONING

1. The argument of sin, cos, exp etc. is dimensionless.
2. Derivative w.r.t. length gives something per length.
3. Dimensions of LHS = Dimensions of RHS

Exercise: Use dimensional reasoning to catch mistakes in the following ~~st~~ statements

$$\frac{\partial}{\partial r}\frac{1}{r} = \ln(r)$$

$$\frac{\partial^2}{\partial x^2}\sin kx = k\cos kx$$

$$\frac{\partial}{\partial x}\sin kx = \frac{1}{k}\sin kx$$

POP QUIZ PH 631 DAY 2

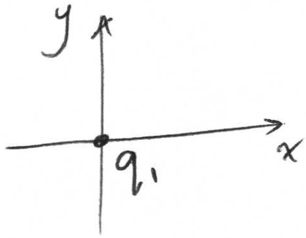
Name ______________________

This question is closed book. Don't look at your notes. But <u>do</u> talk to your classmates.

a) Write down the Poisson Eq^n

b) Derive Gauss's Law

COULOMB POTENTIAL SATISFIES POISSON EQN



$$\Phi(\vec{r}) = \frac{q_1}{4\pi\epsilon_0 |\vec{r}|}$$

$$\rho(\vec{r}) = q_1 \delta(\vec{r})$$

Show that

$$\nabla^2 \Phi = -\frac{\rho(\vec{r})}{\epsilon_0}$$

Our task is to show

$$\nabla^2\left(\frac{q_1}{4\pi\epsilon_0 r}\right) = -\frac{q_1 \delta(\vec{r})}{\epsilon_0}$$

or, equivalently $\nabla^2\left(\frac{1}{4\pi r}\right) = -\delta(\vec{r})$

On day 1 we showed that $\nabla^2\left(\frac{1}{r}\right) = 0$ when $r \neq 0$.

All that remains to show is that

$$\int_{vol} \nabla^2\left(\frac{1}{4\pi r}\right) d^3\vec{r} = -1$$

$$\text{LHS} = \int_{vol} \nabla \cdot \nabla\left(\frac{1}{4\pi r}\right) d^3\vec{r}$$

Stokes thm allows us to avoid the discontinuity at $r=0$.

$$= \int_{surf} \overrightarrow{\nabla\left(\frac{1}{4\pi r}\right)} \cdot d\vec{a}$$

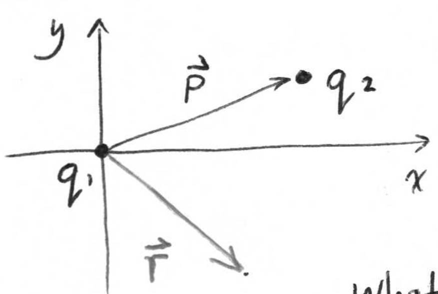
Choose a spherical surface of radius R centered on the origin.

$$= \int_{surf} \frac{-1}{4\pi r^2} \hat{r} \cdot d\vec{a}$$

$$= \frac{-1}{4\pi R^2} 4\pi R^2$$

$$= -1 \qquad \text{Q.E.D.}$$

SUPER POSITION PRINCIPLE



What is Φ at position $\vec{r}$?

$$\boxed{\Phi(\vec{r}) = \frac{q_1}{4\pi\epsilon_0 |\vec{r}|} + \frac{q_2}{4\pi\epsilon_0 |\vec{r}-\vec{p}|}}$$

How do I prove this is true?

Simply check that Poisson Eqn is satisfied.

$$\nabla^2\Phi = \frac{-q_1\delta(\vec{r})}{\epsilon_0} + \frac{-q_2\delta(\vec{r}-\vec{p})}{\epsilon_0} = \frac{-1}{\epsilon_0}\rho(\vec{r}) \quad \checkmark$$

The superposition principle extends to a distribution of 10 charges, 1000 charges, even a continuous distribution of charge.

We will use it again and again.