DAY 27

PH631

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ENERGY OF THE SYSTEM

What do we mean by "energy of the system"?

Working definition

With unpolarized linear dielectric in place we bring in free charges, one by one, allowing the dielectric to respond.

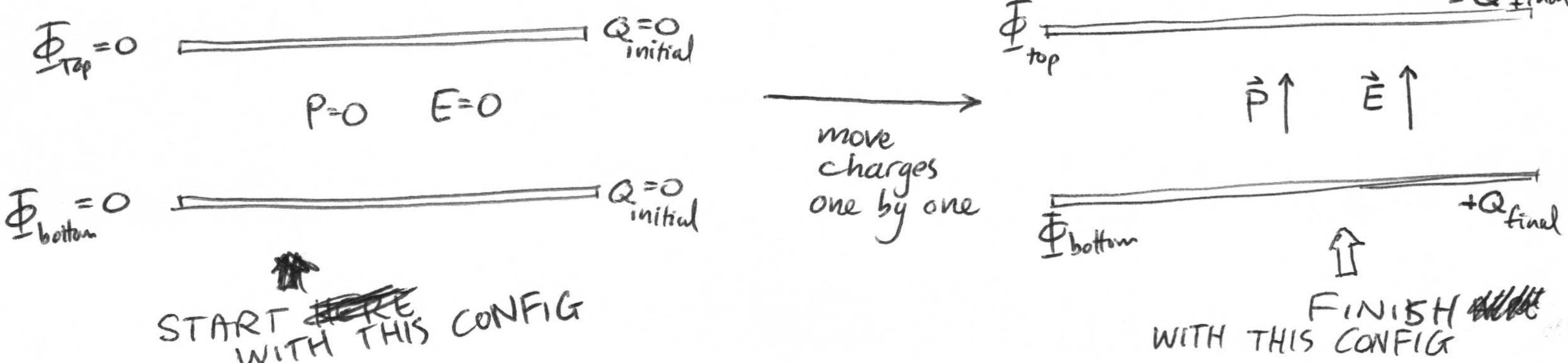
Energy = Work done

The work done pushing each free charge into place.

$$q\Phi(\vec{r}_{final}) - q\Phi(\vec{r}_{initial})$$

This Φ will change every time we move a charge to a new location.

Example



How much work is done getting the system from the starting configuration to the finish?

$$W = \int_{\text{Start}}^{\text{Finish}} dW$$

(work done on each individual charge.)

Define $V = \Phi_{bottom} - \Phi_{Top}$.

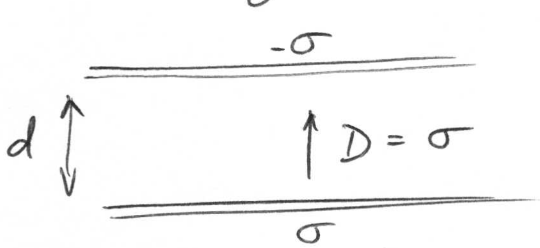
We know $dW = V\,dq$ where V depends on how much charge has accumulated on the top/bottom.

$$V = \frac{Q}{C}$$

$$\Rightarrow \quad W = \int_{\text{Start}}^{\text{Finish}} \frac{Q}{C}\,dQ$$

$$= \frac{1}{2}\frac{Q_{final}^2}{C}$$

The remaining task is to find C.



For linear dielectric $D = \epsilon E$

$$E = \frac{\sigma}{\epsilon}$$ (not ϵ_0)

$$V = Ed = \frac{Qd}{A\epsilon}$$

Capacitance C is the ratio of Q/V

$$\Rightarrow \quad C = \frac{A\epsilon}{d}$$

Therefore, energy stored in a dielectric filled capacitor is

$$\boxed{W = \frac{1}{2}\frac{d}{A\epsilon}Q^2}$$

If we weren't dealing with a capacitor, the strategy of using $W = \int_{\text{Start}}^{\text{Finish}} V\,dQ$ isn't so useful.

There is an alternative way to calculate W

$$W = \frac{1}{2}\int_{\substack{\text{all}\\\text{space}}} \vec{D}\cdot\vec{E}\, d^3\vec{r}$$

Derived in chpt 4 of Griffiths. It is easy to show that this $\vec{D}\cdot\vec{E}$ equation gives the correct result for a dielectric filled capacitor.

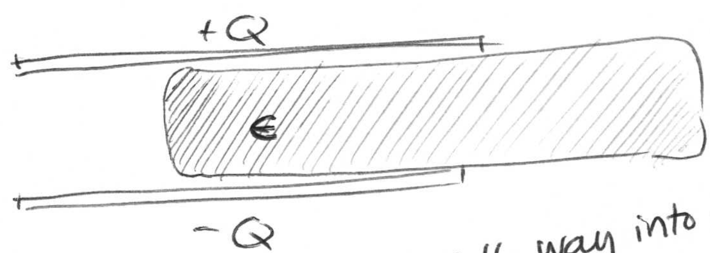
Other applications? I need to look.

Forces on dielectrics

This is a subheading under energy because forces on dielectrics can be computed from position-dependent expressions for energy.

[rather than summing together the force on each individual dipole.]

Example



An insulating dielectric inserted 1/2 way into a charged capacitor will feel a force pushing it further into the capacitor

For qualitative argument, look at the eqn

$$W = \frac{1}{2} \frac{d}{A\epsilon} Q^2$$

W will be lowered when vacuum (ϵ_0) is replaced by dielectric (ϵ).

Now, we will do the exact calculation.