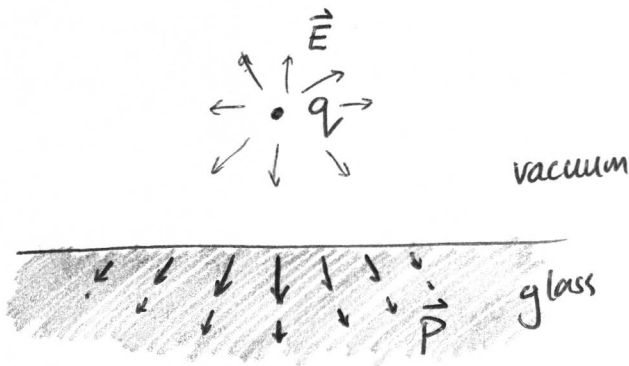


DAY 26

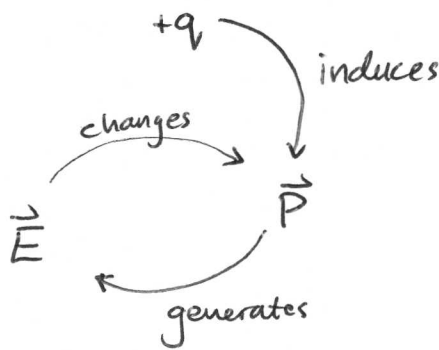
PH 631

Instructor  
Ethan Minot



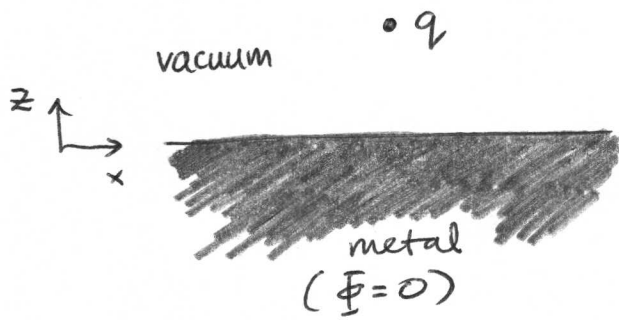
$\vec{E}$ -field and polarization will both be present  
in the glass ( $\vec{P} = \epsilon_0 \chi_e \vec{E}$ )

Looks like a terribly challenging problem to compute  $\vec{P}(\vec{r})$ .



But, we succeeded to solve a similar problem earlier  
in this course: point charge above a flat metal surface.

②



← Boundary conditions

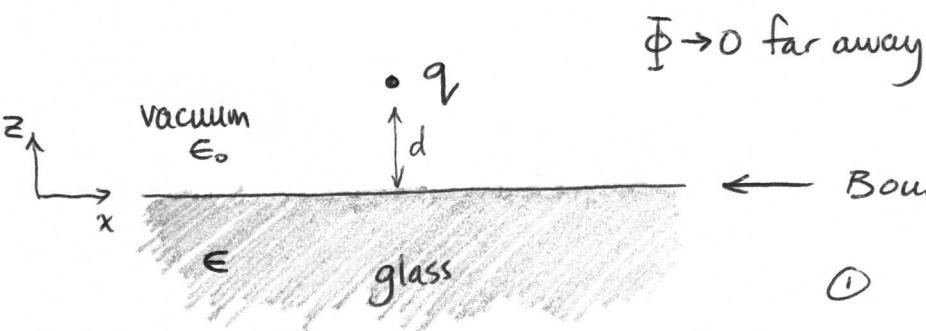
$$\left. \begin{array}{l} \textcircled{1} \quad \Phi = 0 \\ \textcircled{2} \quad \frac{\partial \Phi}{\partial x} = 0 \end{array} \right\} \text{in the plane } z=0$$

We already knew  $\Phi_{\text{below}} = 0$  (potential in grounded metal)

We guessed that 
$$\Phi_{\text{above}} = \frac{q}{4\pi\epsilon_0(x^2+y^2+(z-d)^2)^{3/2}} - \frac{q}{4\pi\epsilon_0(x^2+y^2+(z+d)^2)^{3/2}}$$

We confirmed our guess by checking that BCs were satisfied.

Now, returning to the charge above the glass:



← Boundary conditions

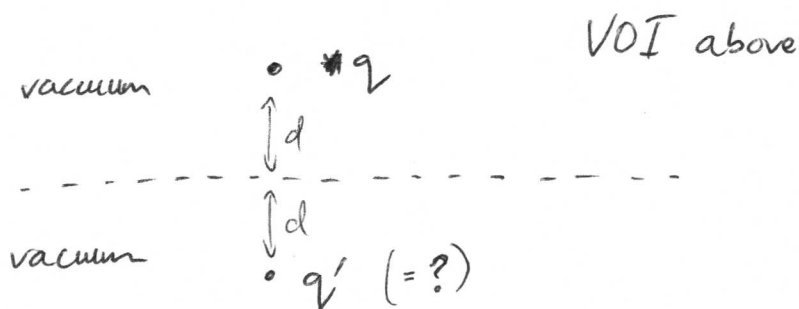
$$\left. \begin{array}{l} \textcircled{1} \quad \Phi_{\text{below}} = \Phi_{\text{above}} \text{ at boundary} \\ \textcircled{2} \quad \epsilon \frac{\partial \Phi_{\text{below}}}{\partial z} \Big|_{z=0} = \epsilon_0 \frac{\partial \Phi_{\text{above}}}{\partial z} \Big|_{z=0} \end{array} \right.$$

(comparing perpendicular components of displacement)

(3)

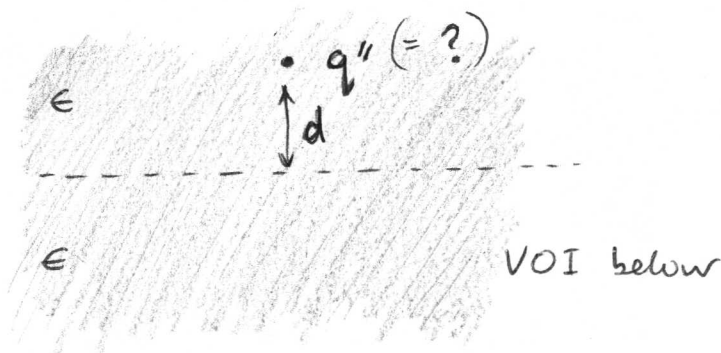
I'll ~~try~~ attempt to ~~recreate~~ <sup>exactly</sup> mimic  $\Phi$  above using

~~using~~



$$\Phi_{\text{above}} = \frac{q}{4\pi\epsilon_0(x^2+y^2+(z-d)^2)^{1/2}} + \frac{q'}{4\pi\epsilon_0(x^2+y^2+(z+d)^2)^{1/2}}$$

I'll attempt to <sup>\*</sup> mimic  $\Phi$  below using



$$\Phi_{\text{below}} = \frac{q''}{4\pi\epsilon(x^2+y^2+(z-d)^2)^{1/2}}$$

To see if we've guessed right, check <sup>whether</sup> the ~~boundary~~ B.C.s are satisfied.

④

yes! they are satisfied when

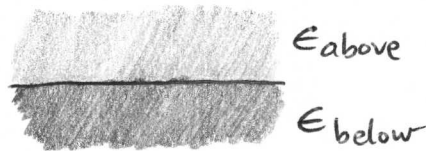
$$q'' = \left( \frac{2\epsilon}{\epsilon + \epsilon_0} \right) q$$

and

$$q' = - \left( \frac{\epsilon - \epsilon_0}{\epsilon + \epsilon_0} \right) q$$

You can check the limits  $\epsilon \rightarrow \epsilon_0$  (no material)  
 $\epsilon \rightarrow \infty$  (slab of metal)

This expression can also be extended  
to any pair of dielectrics by changing  $\epsilon_0 \rightarrow \epsilon_{\text{above}}$   
 $\epsilon \rightarrow \epsilon_{\text{below}}$



Pop quiz: Practice using this result.