DAY 23

PH 631
2015

Instructor
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Last time

$$\nabla^2 \Phi_{tot} = -\frac{(\rho_{free} - \vec{\nabla}\cdot\vec{P})}{\epsilon_0}$$

Just like Poisson eqn, but with an additional source term to account for polarized material.

Massage this eqn.

$$-\vec{\nabla}\cdot\vec{\nabla}\Phi_{tot} = \frac{\rho_{free} - \vec{\nabla}\cdot\vec{P}}{\epsilon_0}$$

$$\epsilon_0 \vec{\nabla}\cdot\vec{E} = \rho_{free} - \vec{\nabla}\cdot\vec{P}$$

$$\boxed{\vec{\nabla}\cdot(\epsilon_0\vec{E} + \vec{P}) = \rho_{free}} \quad \text{—— } (1)$$

This is the first of the four "Maxwell's eqns in Matter"

You will also see it written as

$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho_{free}}$$

where $\vec{D}$ is known as "displacement"

The $\vec{E}$-field in this eqn is generated by both the free charge density $\rho_{free}(\vec{r})$ as well as the dipoles described by $\vec{P}(\vec{r})$.

One strategy for solving Eqn 1 is to choose a ~~g~~ Gaussian surf and integrate

$$\int_{\text{Gaussian surf}} (\epsilon_0 \vec{E} + \vec{P}) \cdot d\vec{a} = \int_{\text{Volume enclosed}} \rho_{free} \, d^3\vec{r} = Q_{free,\, enc}$$

Example, see Pop Quiz

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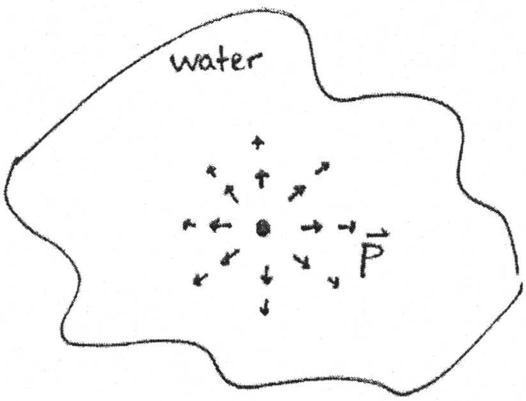
PH631 Pop Quiz, Day ~~22~~ 23

Name:



A positive point charge Q is placed at the origin and surrounded by water (a polarizable medium). The water dipoles point away from Q. The polarization of the water is described by $\mathbf{P}(\mathbf{r})$.

Find the combined quantity $\varepsilon_0\mathbf{E}(\mathbf{r}) + \mathbf{P}(\mathbf{r})$ in the water.

Answer: $\epsilon_0 \vec{E} + \vec{P} = \dfrac{Q}{4\pi r^2}\,\hat{r}$ —— (2)

Most materials are "linear dielectrics", meaning that $\vec{P} \propto \vec{E}$. The constant of proportionality is $\epsilon_0 \chi_e$, where χ_e is a dimensionless number called electric susceptibility.

$$\boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}}$$

for linear dielectrics.

For water $\chi_e = 79$

Plug this into eq^n (2)

$$\epsilon_0 E + \epsilon_0 \chi_e E = \frac{Q}{4\pi r^2}$$

$$E = \frac{Q}{4\pi\epsilon_0 (1+\chi_e)\, r^2}$$

$$= \frac{1}{80}\,\frac{Q}{4\pi\epsilon_0 r^2}$$

This is 80-times smaller that the $\vec{E}$ field around a point charge in vacuum.