

DAY 22

PH 631  
2015

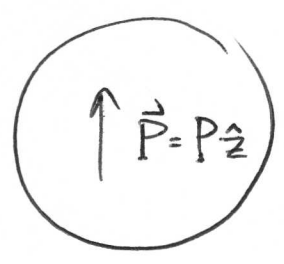
Instructor:  
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# ELECTROSTATICS OF A POLARIZED OBJECT

Practice applying the equation

$$\Phi(\vec{r}) = \int_{\text{surf of object}} \frac{\sigma_b(\vec{r}') d\vec{a}}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|} + \int_{\text{inside object}} \frac{\rho_b(\vec{r}') d^3\vec{r}'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$$

on a sphere that has frozen-in polarization



$$\sigma_b = \vec{P} \cdot \hat{n} = P \cos \theta$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P} = 0$$

Now we know the location of all the bound charge, we can treat the problem like a standard electrostatics problem. Many ~~more~~ solution techniques to choose from. In this particular case, sum of orthogonal fns is the easiest.

For a surface charge we know

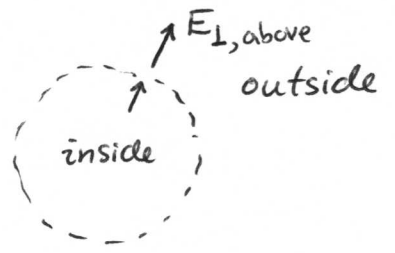
$$E_{1, \text{above}} - E_{1, \text{below}} = \frac{\sigma}{\epsilon_0} \quad \text{--- ①}$$

(2)

From sum of orthogonal functions we can write

$$\Phi_{\text{out}} = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

$$\Phi_{\text{in}} = \sum_{l=0}^{\infty} \underline{A_l} r^l P_l(\cos\theta)$$



To find  $E_{\perp, \text{above}}$  we calculate  $-\frac{\partial \Phi_{\text{out}}}{\partial r} \Big|_{r=R}$

To find  $E_{\perp, \text{below}}$  we calculate  $-\frac{\partial \Phi_{\text{in}}}{\partial r} \Big|_{r=R}$

Plug these results into eq. (1)

$$\sum_{l=0}^{\infty} (l+1) \frac{B_l}{R^{l+2}} P_l(\cos\theta) + \sum_{l=0}^{\infty} l A_l R^{l-1} P_l(\cos\theta) = \frac{1}{\epsilon_0} P \cos\theta$$

↑  
polarization

For equality to hold, LHS must have  $\cos\theta$  dependence

$$\frac{2B_1}{R^3} \cos\theta + A_1 \cos\theta = \frac{P}{\epsilon_0} \cos\theta$$

$$\Rightarrow \boxed{\frac{2B_1}{R^3} + A_1 = \frac{P}{\epsilon_0}}$$

————— (2)

③

We need one more constraint on A, & B,

$$\Phi_{out}(r=R, \theta) = \Phi_{in}(r=R, \theta)$$

$$\frac{B_1}{R^2} \cos\theta = A_1 R \cos\theta$$

$$\boxed{B_1 = A_1 R^3} \quad \text{--- ③}$$

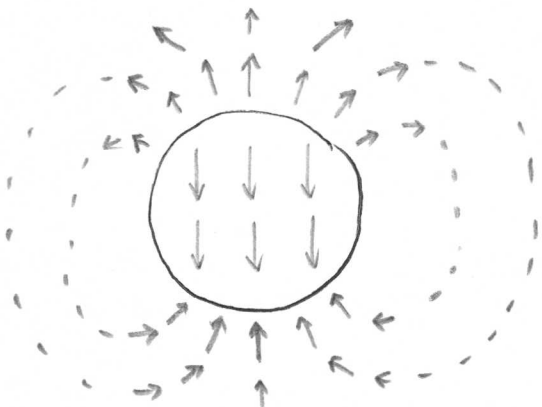
Solving the pair of equations ② & ③ yields

$$A_1 = \frac{P}{3\epsilon_0} \quad \& \quad B_1 = \frac{PR^3}{3\epsilon_0}$$

$$\Phi_{in} = \frac{P}{3\epsilon_0} r \cos\theta = \frac{P}{3\epsilon_0} z$$

[corresponds to  $\vec{E}$ -field pointing in z-direction inside the sphere]

$$\Phi_{out} = \frac{PR^3}{3\epsilon_0} \frac{1}{r^2} \cos\theta = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} \cos\theta \quad \text{where } p = \frac{4}{3}\pi R^3 P$$



corresponds ~~to~~ to the  $\vec{E}$ -field from a perfect dipole centered at the origin.

(4)

## Summary:

Uniformly polarized sphere, equivalent to, shell of charge with  $\sigma = P \cos \theta$ .



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Outside the sphere it looks like a perfect dipole,  
the sum of all the microscopic dipoles.

Inside the sphere,  $\vec{E}$ -field is constant,  $|\vec{E}| = \frac{P}{3\epsilon_0}$ .