

DAY 21

PH 631  
2015

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continued from Day 20

Potential generated by polarized material

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\vec{P}(\vec{r}') \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} d^3\vec{r}'$$

$$= \frac{1}{4\pi\epsilon_0} \int \vec{P}(\vec{r}') \cdot \nabla' \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) d^3\vec{r}'$$

Recall integration by parts for single variable functions

$$\int_{x_1}^{x_2} f(x) \frac{dg(x)}{dx} dx = \left[ f(x)g(x) \right]_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{df(x)}{dx} g(x) dx$$

Applying the vector calculus version yields

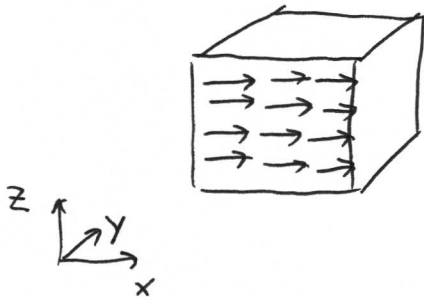
$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left( \left[ \vec{P}(\vec{r}') \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) \right]_{\text{on a surface far far away}} - \int_{\text{volume of all space}} \vec{\nabla}' \cdot \vec{P}(\vec{r}') \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) d^3\vec{r}' \right)$$

goes to zero.

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{all volume}} \frac{-\vec{\nabla}' \cdot \vec{P}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r}'$$

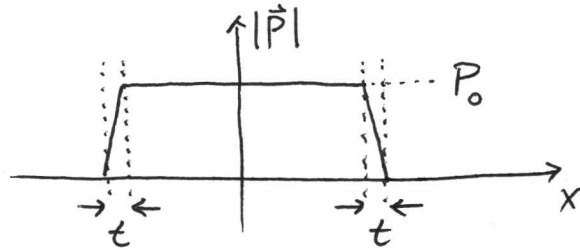
DAY 21

POP QUIZ



Inside the cube,  $\vec{P} = P_0 \hat{x}$

Cross section along x-axis



The thickness  $t$  represents the surface of the material.

a) Calculate the bound charge density,  $-\vec{\nabla} \cdot \vec{P}$ , along the x-axis. Make a plot.

b) Make a 3d sketch of the surfaces where  $-\vec{\nabla} \cdot \vec{P}$  is non-zero. What is the bound charge per unit area on these surfaces?

③

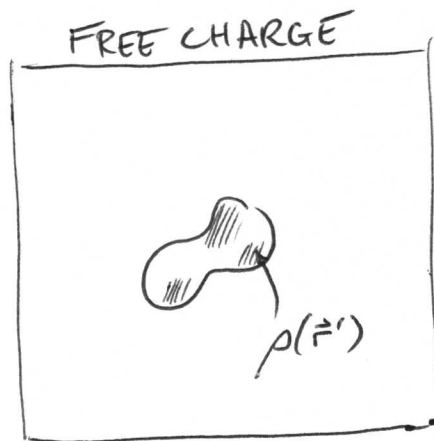
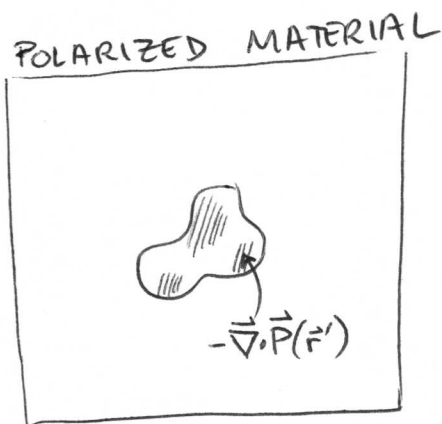
compare to the Green's function integral for ~~for~~ the potential generated by free charge

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{all volume}} \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r}'$$

$-\vec{\nabla} \cdot \vec{P}$  generates potential in the same way as  $\rho$

↑ call this the "bound charge density"

↑ call this the "free charge density"

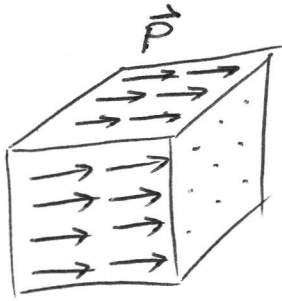


These different physical situations generate the same potential if  $-\vec{\nabla} \cdot \vec{P}(\vec{r}') = \rho(\vec{r}')$ .

Conclusion: Calculating  $\Phi(\vec{r})$  from a polarized material is equivalent to doing a type I electrostatics problem

(4)

~~SURFACE~~ SURFACE BOUNDARY CHARGE DENSITY



On the surface of an object  $\vec{P}(\vec{r}')$  suddenly changes from  $\vec{P}$  to zero.

If the change is a step function, then  $\vec{\nabla} \cdot \vec{P}$  diverges.

Therefore, we typically handle surfaces separately from the bulk of a polarized object.

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left( \int_{\substack{\text{surf} \\ \text{of object}}} \frac{\vec{P}(\vec{r}') \cdot \hat{n}}{|\vec{r} - \vec{r}'|} da + \int_{\substack{\text{volume} \\ \text{inside} \\ \text{object}}} \frac{-\vec{\nabla}' \cdot \vec{P}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r}' \right)$$

$\vec{P}(\vec{r}') \cdot \hat{n}$  is equivalent to a surface charge density.  
 ↖ call this ~~surface~~ "bound surface charge density"

Next: Practice applying these ideas to a sphere with uniform polarization.