DAY 21

PH 631

2015

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continued from Day 20

Potential generated by polarized material

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{P}(\vec{r}\,')\cdot(\vec{r}-\vec{r}\,')\, d^3\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}$$

$$= \frac{1}{4\pi\epsilon_0}\int \vec{P}(\vec{r}\,')\cdot \nabla'\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) d^3\vec{r}\,'$$

Recall integration by parts for single variable functions

$$\int_{x_1}^{x_2} f(x)\frac{dg(x)}{dx}\, dx = \Big[f(x)g(x)\Big]_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{df(x)}{dx} g(x)\, dx$$

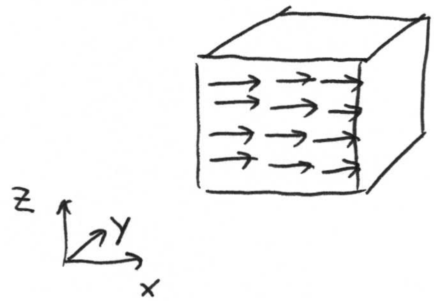
Applying the vector calculus version yields

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\underbrace{\left[\vec{P}(\vec{r})\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right)\right]_{\substack{\text{on a}\\ \text{surface}\\ \text{far far}\\ \text{away}}}}_{\text{goes to zero.}} - \int_{\substack{\text{volume}\\ \text{of all}\\ \text{space}}} \vec{\nabla}\cdot\vec{P}(\vec{r}\,')\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) d^3\vec{r}\,' \right)$$

$$\boxed{\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{\text{all volume}} \frac{-\vec{\nabla}'\cdot\vec{P}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\, d^3\vec{r}\,'}$$

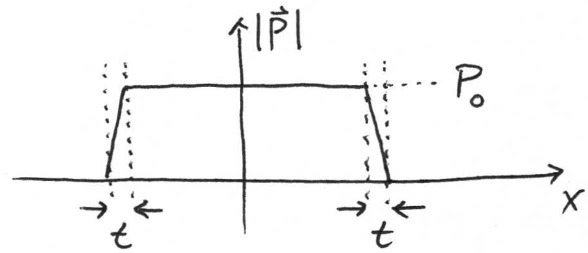
DAY 21

POP QUIZ



Inside the cube, $\vec{P} = P_0 \hat{x}$

Cross section along x-axis



The thickness t represents the surface of the material.

a) Calculate the bound charge density, $-\vec{\nabla} \cdot \vec{P}$, along the x-axis. Make a plot.

b) Make a 3d sketch of the surfaces where $-\vec{\nabla} \cdot \vec{P}$ is non-zero. What is the bound charge per unit area on these surfaces?

compare to the Green's function integral for ~~free ch~~ the potential generated by free charge

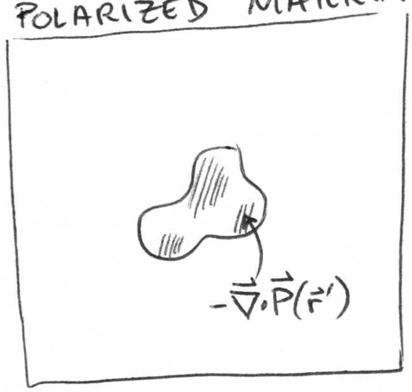
$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\substack{\text{all} \\ \text{volume}}} \frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3\vec{r}\,'$$

$-\vec{\nabla}\cdot\vec{P}$ generates potential in the same way as ρ

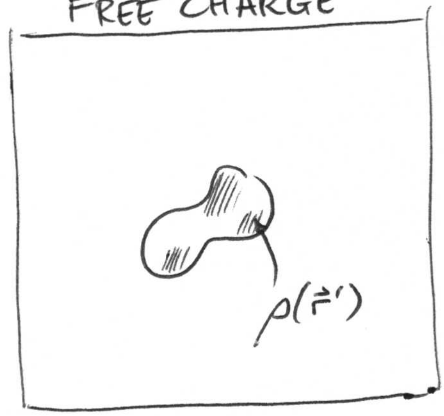
call this the "bound charge density"

call this the "free charge density"

POLARIZED MATERIAL



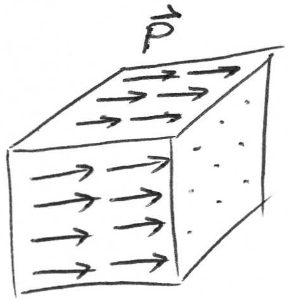
FREE CHARGE



These different physical situations generate the same potential if $-\vec{\nabla}\cdot\vec{P}(\vec{r}\,') = \rho(\vec{r}\,')$.

Conclusion: Calculating $\Phi(\vec{r})$ from a polarized material is equivalent to doing a type I electrostatics problem

~~SURFACE~~ BOUND SURFACE CHARGE DENSITY



On the surface of an object $\vec{P}(\vec{r}')$ suddenly changes from $\vec{P}$ to zero.

If the change is a step function, then $\vec{\nabla}\cdot\vec{P}$ diverges.

Therefore, we typically handle surfaces separately from the bulk of a polarized object.

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\int_{\substack{\text{surf}\\ \text{of object}}} \frac{\vec{P}(\vec{r}')\cdot\hat{n}\, da}{|\vec{r}-\vec{r}'|} + \int_{\substack{\text{volume}\\ \text{inside}\\ \text{object}}} \frac{-\vec{\nabla}'\cdot\vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3\vec{r}' \right)$$

$\underbrace{P(\vec{r}')\cdot\hat{n}}$ is equivalent to a surface charge density.

call this "bound surface charge" density"

Next: Practice applying these ideas to a sphere with uniform polarization.