

DAY 1

2015

PH 631

①

Instructor
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$$\nabla^2 \Phi(\vec{r}) = -\frac{1}{\epsilon_0} \rho(\vec{r})$$

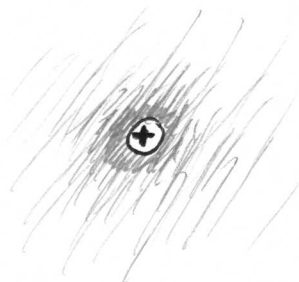
Poisson eqn.

This is the heart of electrostatics.

Learning to wield the Poisson eqn is an exercise in vector calculus that will take several weeks.

Defining the terms

Example



$$\rho(\vec{r}) = q \delta(\vec{r})$$

$$\Phi(\vec{r}) = \frac{q}{4\pi\epsilon_0 r}$$

$$\text{Note } \delta(\vec{r}) = \begin{cases} 0 & r > 0 \\ \text{large} & r = 0 \end{cases}$$

$$\text{How large? } \int_{\text{all space}} \delta(\vec{r}) d^3\vec{r} = 1$$

How do we express the Laplacian, ∇^2 ?

[see the inside front cover of the text book]

a) Show that $\Phi(\vec{r}) = \frac{q}{4\pi\epsilon_0 r}$ is a ~~valid~~ possible solution to the Poisson eqn in regions where $\rho(\vec{r}) = 0$.

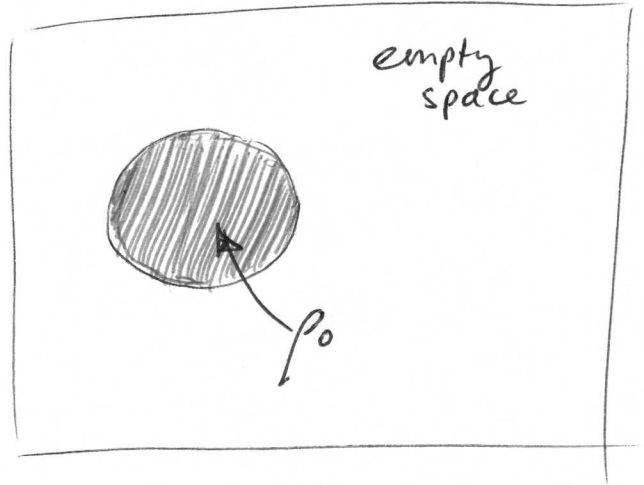
b) Show that $\Phi(\vec{r}) = V_0 \sin kx e^{-kz}$ is also a possible solution to the Poisson eqn in regions where $\rho(\vec{r}) = 0$.

Types of problems in electrostatics

I

$\rho(\vec{r})$ is defined everywhere

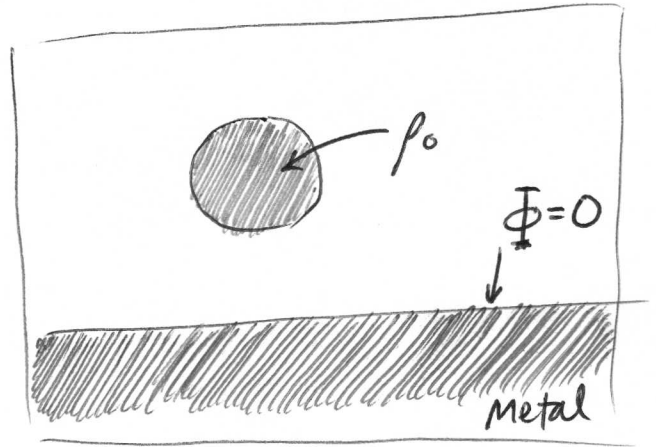
Example



II

$\rho(\vec{r})$ is defined in one region.
 $\Phi(\vec{r})$ is defined on a surface

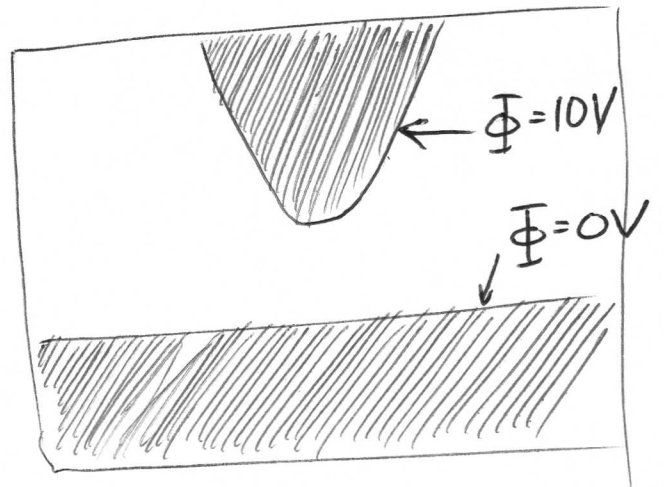
Example



III

$\rho(\vec{r})$ not given.
 $\Phi(\vec{r})$ defined on surfaces

Example



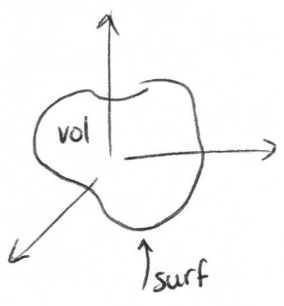
This week we focus on Type I

There are two techniques to choose from

- ① Gauss's Law
- ② Direct integration

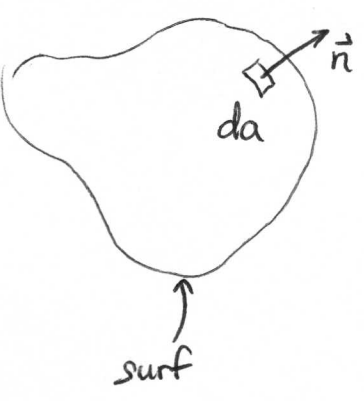
DERIVE GAUSS'S LAW

$$\nabla^2 \Phi = -\frac{\rho}{\epsilon_0} \quad [\text{Poisson eqn}]$$



$$\int_{\text{vol}} \nabla^2 \Phi \, d^3r = \int_{\text{vol}} -\frac{\rho}{\epsilon_0} \, d^3r$$

$$\int_{\text{vol}} \nabla \cdot \nabla \Phi \, d^3r = -\frac{Q_{\text{enc}}}{\epsilon_0}$$



$$\int_{\text{surf}} \nabla \Phi \cdot \vec{n} \, da = -\frac{Q_{\text{enc}}}{\epsilon_0}$$

note that $\vec{E} = -\nabla \Phi$

$$\int_{\text{surf}} \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

↑
short hand
for $\vec{n} \, da$

Gauss's Law is another way of expressing the Poisson eqn.