Instructor:
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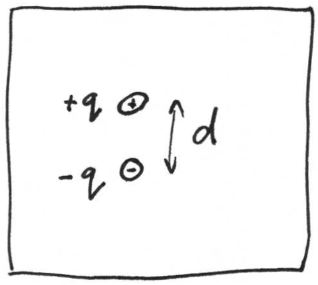
MULTIPOLE EXPANSION CONTINUED...

Last time



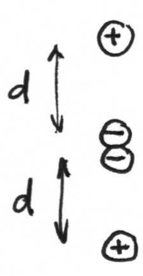
$$\Phi(r,\theta) = \frac{qd}{4\pi\epsilon_0}\frac{\cos\theta}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

By putting two opposing monopoles close to each other we killed the $\frac{1}{r}$ term at large r.

Now, lets put two opposing dipoles close to each other

Find $\Phi(\vec{r})$ at large r

<u>Step 1:</u> Calculate Φ on z-axis for large z.



$$\Phi(0,0,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{-2q}{z} + \frac{q}{z+d} + \frac{q}{z-d}\right)$$

$$= \frac{q}{4\pi\epsilon_0 z}\left(-2 + \frac{1}{1+d/z} + \frac{1}{1-d/z}\right)$$

in the limit of large z

$$= \frac{q}{4\pi\epsilon_0 z}\left(2 + \left(1 - \frac{d}{z} + \frac{d^2}{2z^2} + \dots\right) + \left(1 + \frac{d}{z} - \frac{d^2}{2z^2} + \dots\right)\right)$$

$$= \frac{q}{4\pi\epsilon_0 z}\left(\frac{d^2}{z^2} + \mathcal{O}\left(\frac{1}{z^3}\right)\right)$$

$$= \frac{qd^2}{4\pi\epsilon_0 z^3} + \mathcal{O}\left(\frac{1}{z^4}\right) \qquad \text{——— } (1)$$

Step 2: Use this as a boundary condition to match to the sum of orthogonal fns in spherical coordinates (seperable fns that satisfy $\nabla^2\Phi = 0$).

$$\Phi(r,\theta) = \frac{B_0}{r}P_0(\cos\theta) + \frac{B_1}{r^2}P_1(\cos\theta) + \frac{B_2}{r^3}P_2(\cos\theta) + \cdots$$

$$\Rightarrow \underbrace{\Phi(r, \theta=0)}_{\text{along the } z \text{ axis}} = \frac{B_0}{z}P_0(1) + \frac{B_1}{z^2}P_1(1) + \frac{B_2}{z^3}P_2(1) + \cdots$$

comparision with eq^n (1)

$$\Rightarrow B_2 = \frac{qd^2}{4\pi\epsilon_0} \qquad B_1 = 0 \qquad B_0 = 0$$

Final answer:

$$\Phi(r,\theta) = \underbrace{\frac{qd^2}{4\pi\epsilon_0 r^3}P_2(\cos\theta)}_{\text{This is the quadropole term in the multipole expansion.}} + \mathcal{O}\left(\frac{1}{r^4}\right)$$