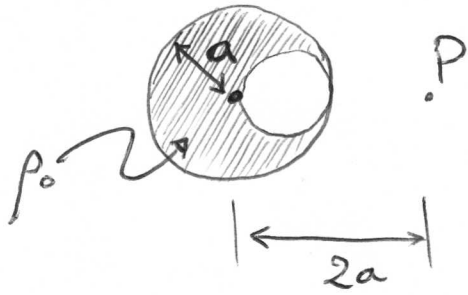


DAY 10

PH 631

Instructor: Ethan Minot

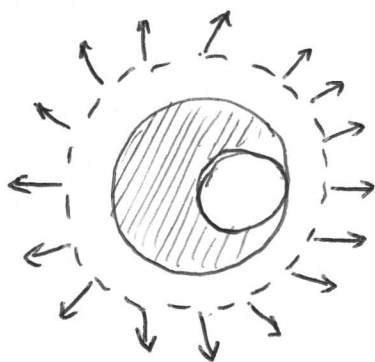
BLACK BOARD QUESTION



A sphere of radius "a" has charge density ρ_0 and ~~a hollow~~ except for a hollow cavity. The hollow cavity has radius $a/2$ and is offset from the center of the sphere.

Find \vec{E} at point P.

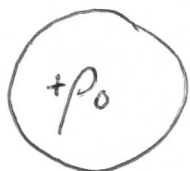
Solution



We cannot apply Gauss's Law directly because \vec{E} is not uniform and not radial across a spherical Gaussian surface.

Therefore, we break the problem into two parts

Part I



P

$$\vec{E} = \frac{+Q}{4\pi\epsilon_0(2a)^2} \hat{r}$$

Part II

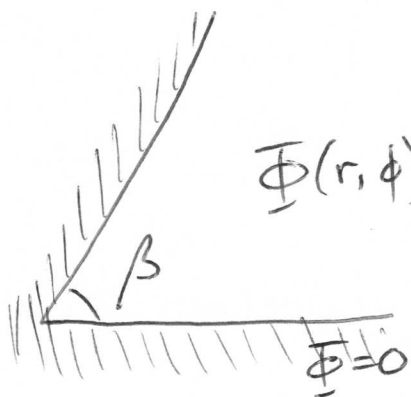


P

$$\vec{E} = \frac{-Q/8}{4\pi\epsilon_0\left(\frac{3a}{2}\right)^2} \hat{r}$$

Then we superimpose the two charge distributions and the two \vec{E} fields.

Continuing summation of orthogonal functions from Day 9.



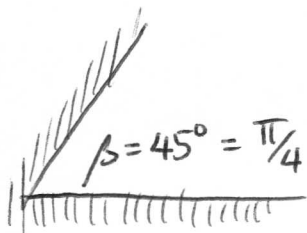
$$\Phi(r, \phi) = \sum_{\nu} A_{\nu} r^{\nu} \sin \nu \phi$$

when $\phi = \beta$ this term must go to zero

$$\Rightarrow \nu/\beta = n\pi \quad (n=1, 2, 3, \dots)$$

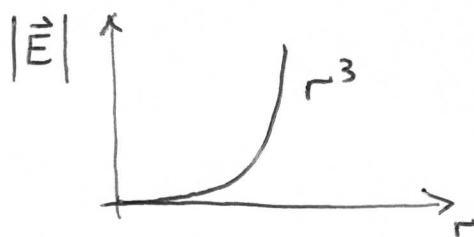
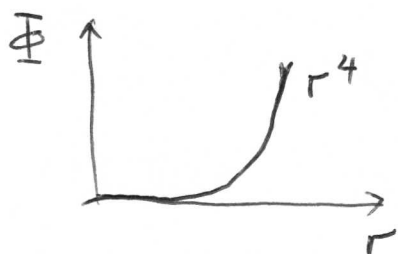
$$\begin{aligned} \Phi(r, \phi) &= \cancel{\sum_{\nu} A_{\nu} r^{\nu} \sin \nu \phi} \sum_{n=1}^{\infty} A_n r^{n\pi/\beta} \sin \frac{n\pi}{\beta} \phi \\ &= A_1 r^{\pi/\beta} \sin \frac{\pi}{\beta} \phi + A_2 r^{2\pi/\beta} \sin \frac{2\pi}{\beta} \phi + \dots \end{aligned}$$

Examples

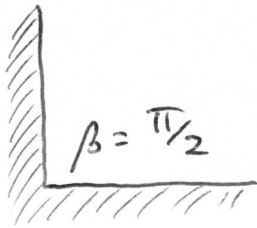


$$\Phi = A_1 r^4 \sin 4\phi + A_2 r^8 \sin 8\phi + \dots$$

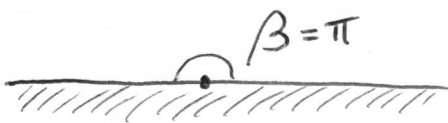
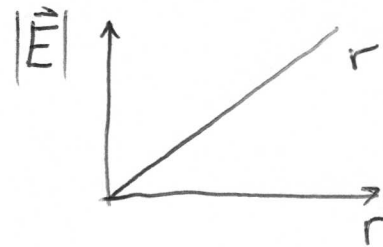
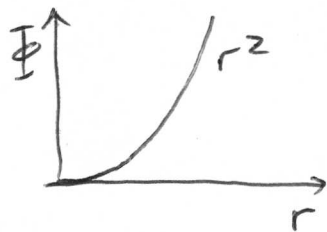
This term dominates at small r .



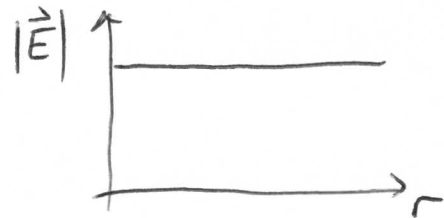
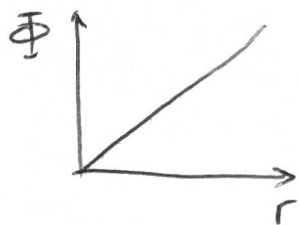
The corner is protected, \vec{E} does not get into corner.



$$\Phi = A_1 r^2 \sin 2\phi + \dots$$



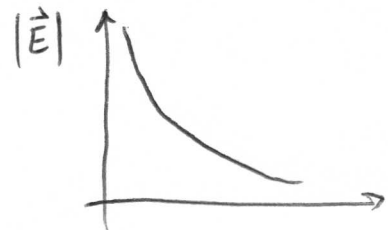
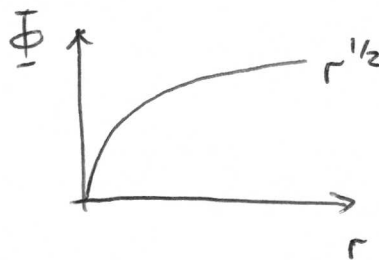
$$\Phi = A_1 r \sin \phi + \dots$$



constant $|\vec{E}|$ above a flat metal surface.



$$\Phi = A_1 r^{1/2} \sin \phi/2 + \dots$$

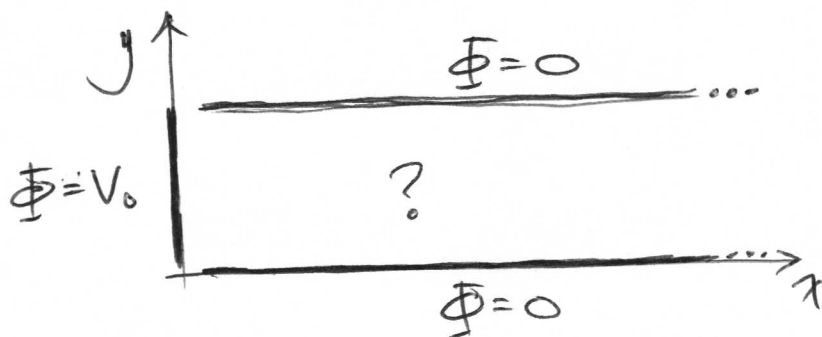


The $|\vec{E}|$ -field diverges near the tip.

SUMMARY: The power-law dependence of potential Φ near a sharp corner is sensitive to β . ^{vs. position}

Applications of summation of orthogonal functions.

Example 2



Step 1: Choose cartesian coordinates, therefore

$$\begin{aligned}\Phi(x,y) = & \sum_{\alpha} A_{\alpha} \sin \alpha x e^{-\alpha y} + \sum_{\beta} A_{\beta} \sin \beta y e^{-\beta x} \\ & + \sum_{\gamma} A_{\gamma} \cos \gamma x e^{-\gamma y} + \sum_{\xi} A_{\xi} \cos \xi y e^{-\xi x}\end{aligned}$$

Step 2: Use B.C.s to decide which terms we need to keep in the summation.

$$\Phi \rightarrow 0 \text{ as } x \rightarrow \infty$$

$$\Rightarrow A_{\alpha} = 0 \text{ and } A_{\gamma} = 0$$

$$\Phi = 0 \text{ when } y = 0$$

$$\Rightarrow A_{\xi} = 0$$

$$\therefore \Phi(x,y) = \sum_{\beta} A_{\beta} \sin \beta y e^{-\beta x}$$