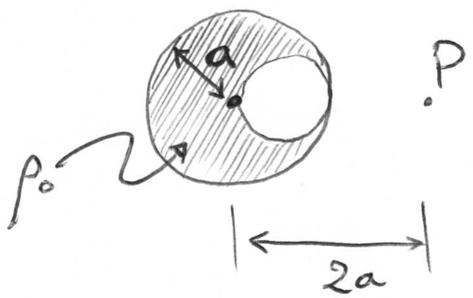
DAY 10

PH 631

Instructor: Ethan Minot

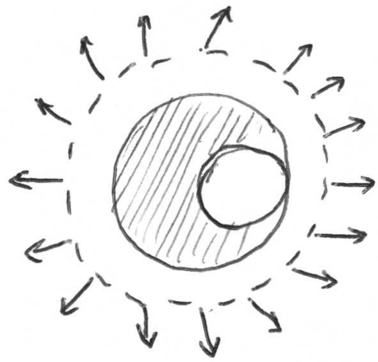
BLACK BOARD QUESTION



A sphere of radius "a" has charge density ρ_0 ~~and a hollow~~ except for a hollow cavity. The hollow cavity has radius $a/2$ and is offset from the center of the sphere.

Find $\vec{E}$ at point P.

Solution



We cannot apply Gauss's Law directly because $\vec{E}$ is not uniform and not radial across a spherical Gaussian surface.

Therefore, we break the problem into two parts

Part I

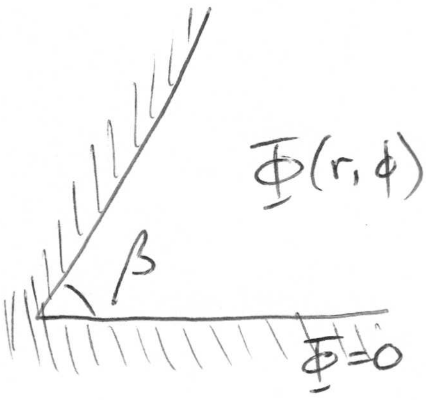
$$\vec{E} = \frac{+Q}{4\pi\epsilon_0 (2a)^2}\,\hat{r}$$

Part II

$$\vec{E} = \frac{-Q/8}{4\pi\epsilon_0 \left(\frac{3a}{2}\right)^2}\,\hat{r}$$

Then we superimpose the two charge distributions and the two $\vec{E}$ fields.

Continuing summation of orthogonal functions from Day 9.



$$\Phi(r,\phi) = \sum_{\nu} A_\nu r^\nu \underbrace{\sin \nu\phi}$$

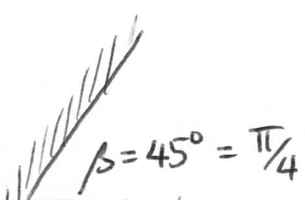
when $\phi=\beta$ this term must go to zero

$$\Rightarrow \nu\beta = n\pi \quad (n=1,2,3\ldots)$$

$$\Phi(r,\phi) = \sum_{n=1}^{\infty} A_n r^{n\pi/\beta} \sin \frac{n\pi}{\beta}\phi$$

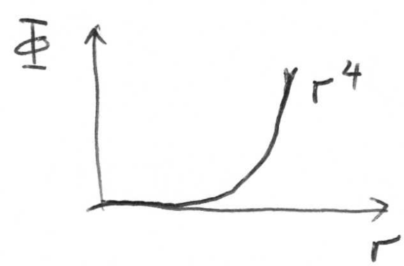
$$= A_1 r^{\pi/\beta} \sin \frac{\pi}{\beta}\phi + A_2 r^{2\pi/\beta} \sin \frac{2\pi}{\beta}\phi + \ldots$$

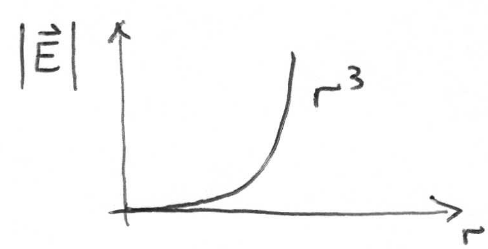
Examples



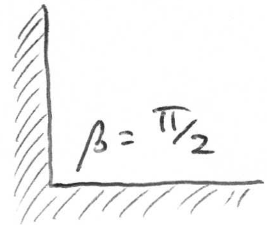
$$\Phi = \underbrace{A_1 r^4 \sin 4\phi} + A_2 r^8 \sin 8\phi + \ldots$$

This term dominates at small r.

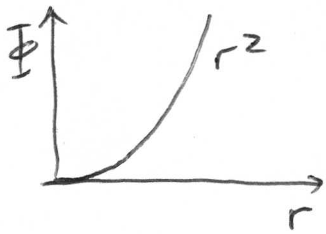


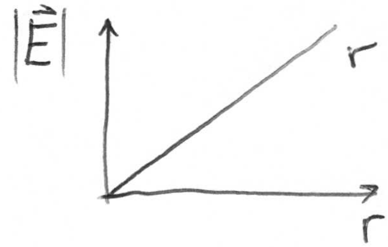


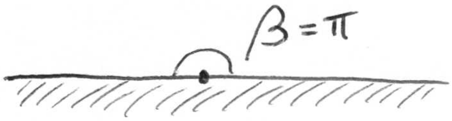
The corner is protected, $\vec{E}$ does not get into corner.



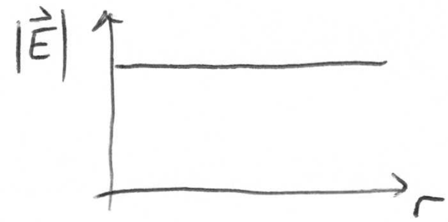
$$\Phi = A_1 r^2 \sin 2\phi + \ldots$$



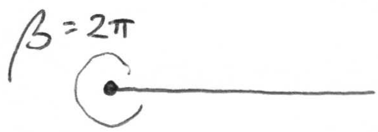




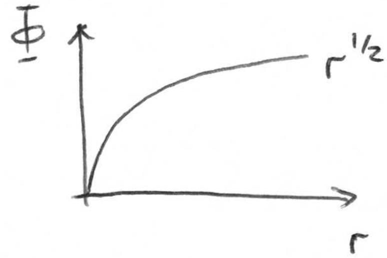
$$\Phi = A_1 r \sin\phi + \ldots$$

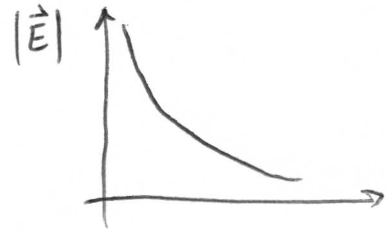


Constant $|\vec{E}|$ above a flat metal surface.



$$\Phi = A_1 r^{1/2} \sin \phi/2 + \ldots$$



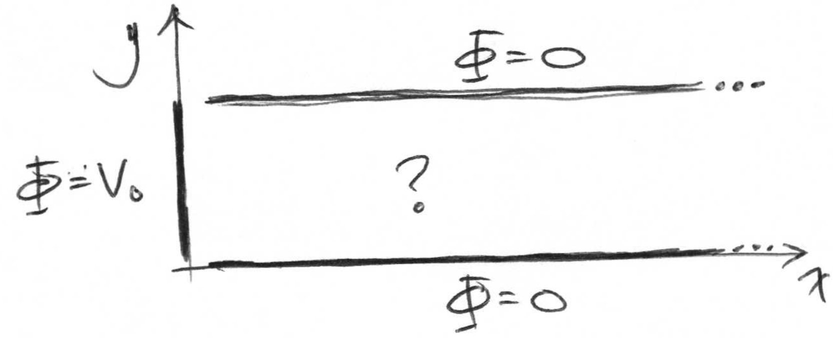


The $|\vec{E}|$-field diverges near the tip.

<u>SUMMARY</u>: The power-law dependence of potential vs. position near a sharp corner is sensitive to β.

Applications of summation of orthogonal functions.

Example 2



Step 1: Choose cartesian coordinates, therefore

$$\Phi(x,y) = \sum_\alpha A_\alpha \sin\alpha x\, e^{-\alpha y} + \sum_\beta A_\beta \sin\beta y\, e^{-\beta x}$$

$$+ \sum_\gamma A_\gamma \cos\gamma x\, e^{-\gamma y} + \sum_\xi A_\xi \cos\xi y\, e^{\xi x}$$

Step 2: Use B.C.s to decide which terms we need to keep in the summation.

$\Phi \to 0$ as $x \to \infty$

$\Rightarrow A_\alpha = 0$ and $A_\gamma = 0$

$\Phi = 0$ when $y = 0$

$\Rightarrow A_\xi = 0$

$$\therefore \quad \Phi(x,y) = \sum_\beta A_\beta \sin\beta y\, e^{-\beta x}$$