

PH561

DAY 5

HOMEWORK #2 HINTS

ON YOUR
BOARDS

Find the phase difference between

$$\frac{2}{1+i} e^{-i\omega t}$$

and $\frac{8}{1-i} e^{-i\omega t}$

RECIPE FOR APPLYING THE GREEN'S FN TECHNIQUE

- ① Identify the ^{inhomogeneous} D.E. that your unknown function must satisfy

$$\mathcal{L} u(x) = f(x)$$

Linear differential operator

Make note of any boundary conditions that $u(x)$ must satisfy.

- ② Find the Green's function that is unique to the particular \mathcal{L} and the particular boundary conditions in your problem.

$$\mathcal{L} G(x, \xi) = \delta(x - \xi)$$

where $G(x, y)$ satisfies the same B.C.s as the unknown fn $u(x)$.

Note that the RHS = 0 when

$x < \xi$ & $x > \xi$ so the Green's fn will likely splice together solns of the homogeneous eqn.

- ③ Find $u(x)$ by calculating the integral

$$u(x) = \int_a^b \underbrace{f(\xi) d\xi}_{\text{called an impulse}} G(x, \xi)$$

if it has dimensions Force · Time

Note: Integrate over ξ to leave a fn of x .

①

GREEN'S FUNCTION TECHNIQUE (CONTINUED)

Last time:

Apply Green's function technique to solve

$$\frac{d^2 u}{dx^2} = \frac{F(x)}{T} \quad \text{where } u=0 \text{ at } x=0, x=L.$$

$$(\mathcal{L}u = f(x))$$

STEP 1: Find Green's fn, i.e. $G(x, \xi)$ s.t.

$$\mathcal{L}G = \frac{d^2 G}{dx^2} = \delta(x - \xi)$$

we got to the point

$$G = \begin{cases} \frac{A}{\xi} x & x < \xi \\ \frac{A(L-x)}{(L-\xi)} & x > \xi \end{cases}$$

i.e. Solving
the homogeneous
eqn to the
left and right
of $x = \xi$.

to complete step 1, we need to find A.

② ●

~~Find~~ set A such that

$$\frac{d^2 G}{dx^2} = \delta(x - \xi)$$

We do this by integrating over the δ -fn

$$\int_{\xi - \epsilon}^{\xi + \epsilon} \frac{d^2 G}{dx^2} = \int_{\xi - \epsilon}^{\xi + \epsilon} \delta(x - \xi)$$

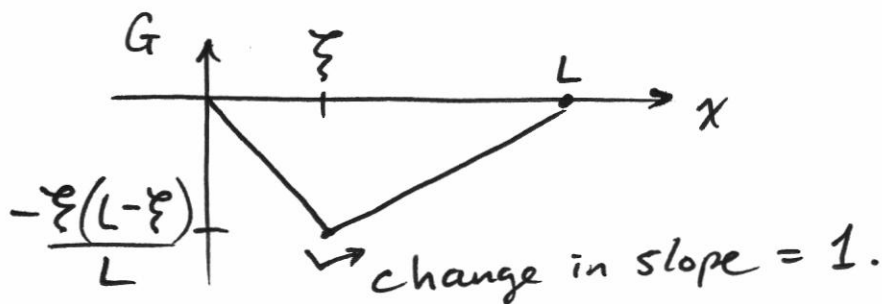
$$\left. \frac{dG}{dx} \right|_{x=\xi+\epsilon} - \left. \frac{dG}{dx} \right|_{x=\xi-\epsilon} = 1$$

$$\left(\frac{-A}{L - \xi} \right) - \frac{A}{\xi} = 1$$

$$\Rightarrow A = \frac{-\xi(L - \xi)}{L}$$

Plug this back into the $G(x, \xi)$ eqn to find

$$G(x, \xi) = \begin{cases} -\frac{x(L - \xi)}{L} & , x < \xi \\ -\frac{\xi(L - x)}{L} & , x > \xi \end{cases}$$



3

Now we know the Green's fn, we can use it to find the curve of the string under any load ~~pattern~~ distribution (i.e. changing the source term in the DE).

Example

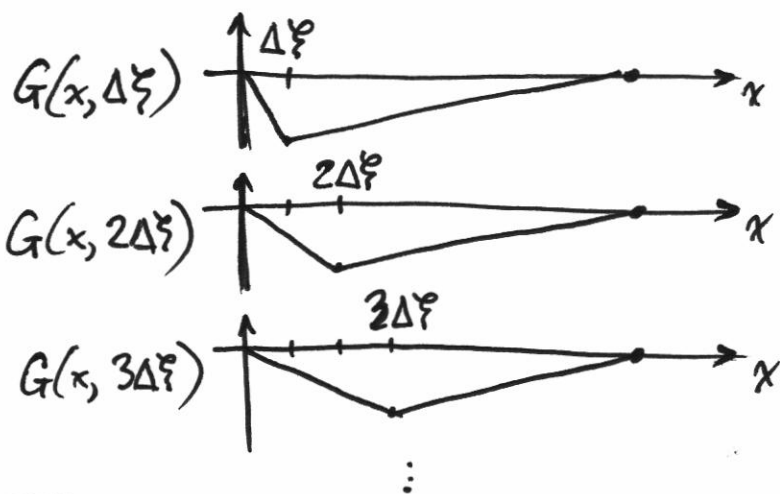


$$\frac{d^2u}{dx^2} = \frac{F_0}{T} \quad (\text{same force at every position}).$$

~~1.1~~ Solution using a computer.

1. Choose a small $\Delta\xi$

2. Evaluate $G(x, \xi)$ for $\xi = 0, \Delta\xi, 2\Delta\xi, \dots, L$



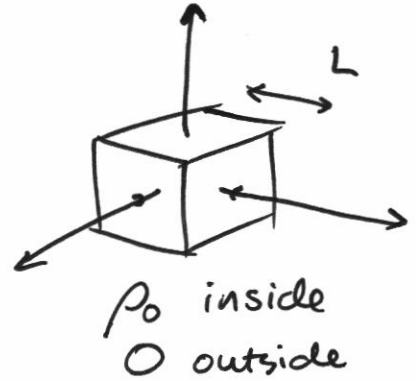
3.

A graph showing the total displacement $u(x)$ versus x . The curve is a smooth, downward-opening parabola-like shape, symmetric about $x=L/2$. The equation is given as:

$$u(x) = \sum_{i=1}^N G(x, \xi_i) F_0 \Delta\xi$$

GREEN'S FUNCTION FOR SOLVING PDES

Example: $\nabla^2 \phi = \frac{\rho_{\text{cube}}(x, y, z)}{\epsilon_0}$



B.C. $\phi = 0$ at infinity.

Solution: (Follow the recipe)

Step 1: $\mathcal{L} = \nabla^2$

Step 2: $\nabla^2 G(\vec{r}, \vec{\xi}) = \delta(\vec{r} - \vec{\xi})$, find G.

Try $G(\vec{r}, \vec{\xi}) = \frac{1}{|\vec{r} - \vec{\xi}|}$ (it is consistent with the B.C.)

Does $\nabla^2 G = 0$ when $\vec{r} \neq \vec{\xi}$?

↑
Please check this explicitly using Cartesian coordinates.