EXAMPLE EXAM QUESTIONS COVERING DAY 18 - 25.

① Calculate the Fourier transform of $\delta(x)$.

② Using the special property of Hermitian differential operators

$$\int_a^b \left(\hat{L} f(x)\right)^* g(x)\, dx = \int_a^b (f(x))^* \hat{L}\, g(x)\, dx$$

Show that the eigenvalues of a Hermitian operator are real.

③ Starting from the three linearly independent functions

$$1,\ e^x,\ e^{-x}$$

Construct 3 orthonormal functions over the interval $[-1, 1]$. Do these three functions form a complete basis?

④ a) Put the Hermite eq. into Sturm-Louiville form. The Hermite eq. is

$$y'' - 2xy' + 2\lambda y = 0 \qquad [-\infty, \infty]$$

b) How would you define the inner product so that the eigenfns of the Hermite eq" are orthogonal ?

MISCILLANEOUS QUESTIONS:

⑤ Sketch the function

$$y(x) = (x - \pi)\sin x$$

over the range 0 to 2π.

Pay attention to the curvature, the zeros, the scale of the y-axis.

⑥ Use dimensional analysis to find the scaling relation between the orbital period of a planet and its orbital radius. Assume the mass of the planet is much smaller than the mass of the sun.

Note, the gravitational constant G has dimensions $\dfrac{\text{Force} \cdot \text{Length}^2}{\text{Mass}^2}$