Announcements: If you lost points on hw, it is possible ~~to~~ that you will get some back. Discuss your reasoning with Justyna.

Mahajan

1. Dimensions
2. Easy cases
3. Lumping
4. Pictorial proofs
5. Taking out the big part

Today's topic (wrapping up Mahajan material)

FIRST: Approximate and understand the most important effect in a physical problem

SECOND: Refine the analysis and understanding.

"Doing first things first"

As a metaphor for deeper physics problems, start with a technique used for quick mental arithmatic.

$$3.15 \times 7.21 = ?$$

The big part: $3 \times 7 = 21$

Refining the answer

$$3.15 = 3(1 + 0.05)$$
$$7.21 = 7(1 + 0.03)$$

Refined estimate

$$21(1 + 0.08)$$

I added the fractional changes.

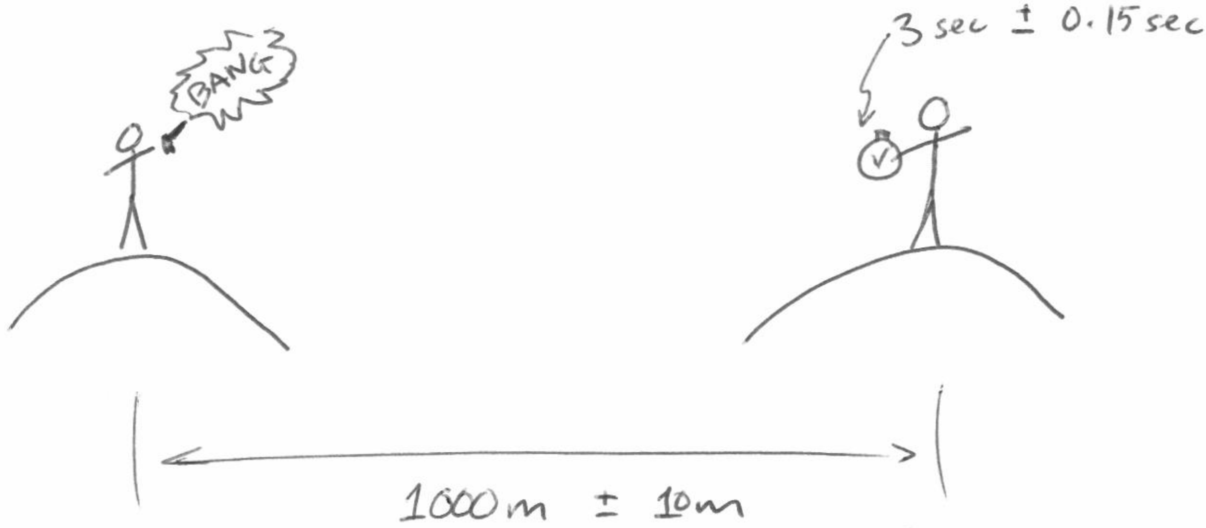
$$= 22.7$$

Adding small fractional changes is a very useful tool and deserves some discussion.

$$(x + \Delta x)(y + \Delta y) = x\left(1 + \frac{\Delta x}{x}\right) y \left(1 + \frac{\Delta y}{y}\right)$$

$$= xy\left(1 + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \underbrace{\frac{\Delta x \Delta y}{xy}}_{\text{2nd order correction.}}\right)$$

In experimental physics, this concept is used for error analysis



What is the uncertainty in the measured speed of sound?

$$\frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x\left(1 \pm \frac{\Delta x}{x}\right)}{y\left(1 \pm \frac{\Delta y}{y}\right)} \approx \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \mp \frac{\Delta y}{y}\right)$$

highest value $\frac{x}{y}\left(1 + \frac{\Delta x}{x} + \frac{\Delta y}{y}\right)$

lowest value $\frac{x}{y}\left(1 - \frac{\Delta x}{x} - \frac{\Delta y}{y}\right)$

- Small fractional changes in <u>multiplied</u> quantities add together.

- Small fractional changes in <u>divided</u> quantities are subtracted.

What about a squared quantity?

<u>Example</u>: Driving to Portland at 65 mph takes a ½ tank of gas. How much would be needed if you drove at 55 mph?

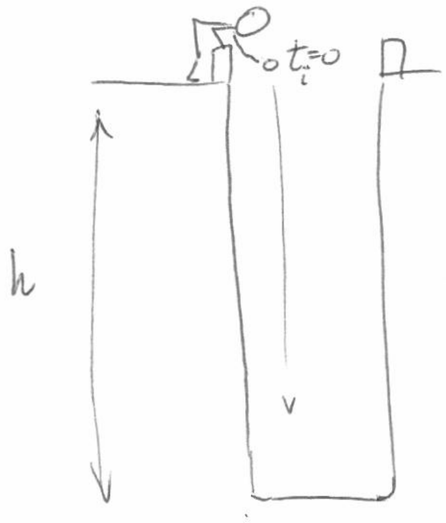
Assume gas consumption is dominated by drag force

$$\text{Work done} = F\,d$$

$$\propto v^2 d$$

Drop speed by 15% $\Rightarrow$ Drop work done by $\boxed{}$

Now apply "first things first" to a physics problem



h is unknown.

But ~~we hear~~ a sound is heard at $t^* = 4$ sec.

Find h.

Assume $g = 10\ \mathrm{ms^{-2}}$

$c_s = 340\ \mathrm{ms^{-1}}$

The precise method involves solving a quadratic eq^n.

$$t_{fall} = \sqrt{\frac{2h}{g}} \qquad t_{sound} = \frac{h}{c_s}$$

$$t^* = \sqrt{\frac{2h}{g}} + \frac{h}{c_s} \qquad \text{solve for } h$$

$$h = \left(\sqrt{t^* c - \frac{c_s^2}{2g}} - \frac{c_s}{2}\sqrt{\frac{2}{g}} \right)^2 = 71.56\ \mathrm{m}$$

"A triumpf of symbol manipulation!"

"A high entropy expression"

Now the "first things first approach"

- The rock travels much slower than c_s, therefore, the zeroth order approx is

$$h \approx \frac{g(t^*)^2}{2} = 80\text{m}$$

- Now refine the answer [work in pairs]

Note that the refinement can be iterated

$$h_i = \frac{g}{2}\left(t^* - \frac{h_{i-1}}{c_s}\right)^2$$

previous approx.

better approx

$\boxed{\text{LOW ENTROPY EXPRESSION}}$