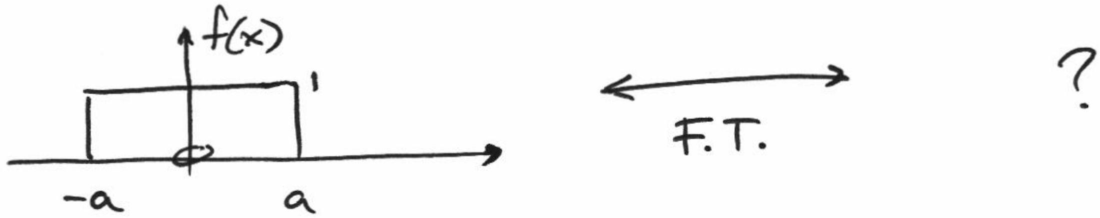
FOURIER TRANSFORMS (continued)

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) e^{-ikx} dk$$

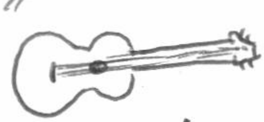
$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{ikx} dx$$

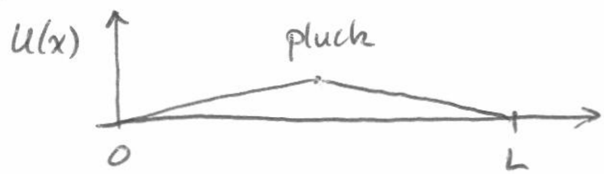
§7.1 Butkov

Return to the top hat example

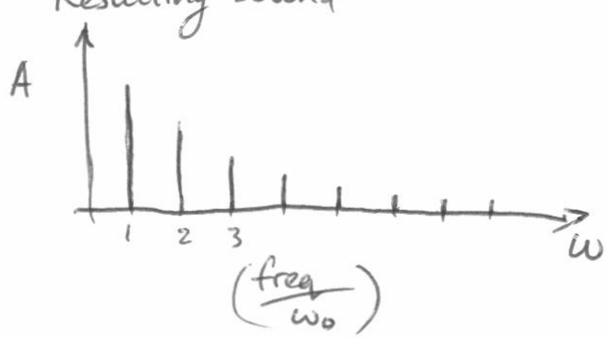


Review HW5, question #2.

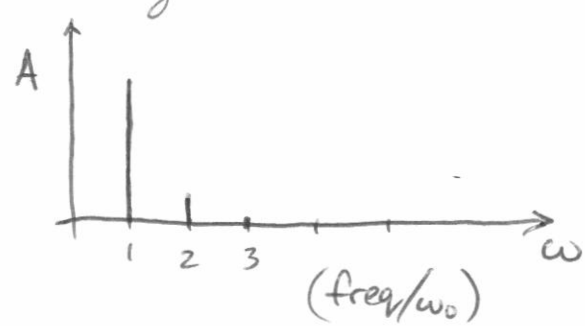




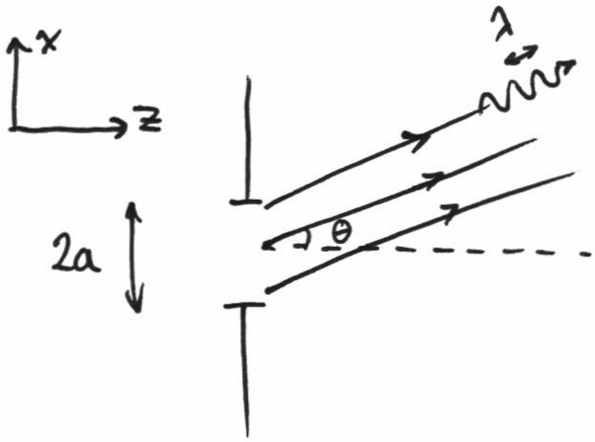
Resulting sound



Resulting sound



Relationship to diffraction patterns in Optics:



Far field
Intensity as function of θ.

$$I = I_0 \left(\frac{\sin Ka}{Ka} \right)^2$$

where $K = \frac{2\pi}{\lambda} \sin\theta$

$$f(x) = \begin{cases} 1 & -a < x < a \\ 0 & |x| > a \end{cases} \quad \xleftrightarrow{FT} \quad |F(k)|^2$$

$f(x)$ corresponds to the ~~apateur~~ aperture.

$|F(k)|^2$ corresponds to the diffraction pattern on the screen.

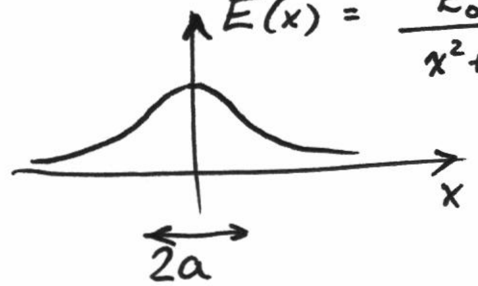
In this case, k is equivalent to the x-component of the photon momentum.

Relationship to the Uncertainty Principle in QM:

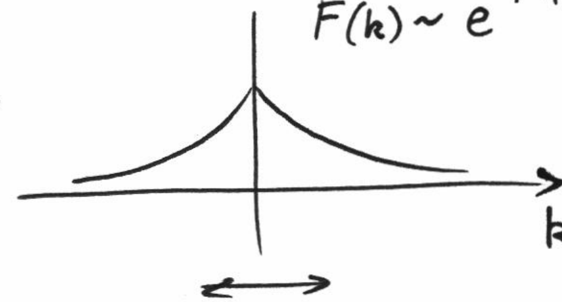
The FT shows the spatial frequencies contained in a wave packet.

For example [Butkov 7.2, example 2]

$$E(x) = \frac{E_0}{x^2 + a^2} \quad \Longleftrightarrow \quad F(k) \sim e^{-|k|a}$$



FWHM
(Full width at half max)



$\approx 2/a$ FWHM

For any wave packet, you will find

$$\Delta x \Delta k \approx 4$$

"UNCERTAINTY PRINCIPLE FOR WAVE PACKETS" follows from F.T.

Recall from QM, the momentum of a particle/wave is $p = \hbar k$.

$$\Rightarrow \Delta k = \frac{\Delta p}{\hbar}$$

$$\Delta x \frac{\Delta p}{\hbar} \approx 1 \quad \text{c.f.} \quad \boxed{\Delta x \Delta p \gtrsim \frac{\hbar}{2}}$$

UNCERTAINTY PRINCIPLE

In this case Δx & Δp are standard deviations.

Important general knowledge ~~[crossed out]~~
(might even be useful in Trivial Pursuits)

Q: What function will keep the same ~~[crossed out]~~ shape after a Fourier Transform?

Butkov §7.2 Example 1 has the exact pair of fns.

We can guess the pair of functions using the "wavepacket uncertainty principle".

Q: What is the FT of a δ-fn?

$$\int_{-\infty}^{\infty} F(k)\, e^{ikx}\, dk = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{ikx}\, dk$$

$$= 2\int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} \cos kx \, dk$$

$$\left[\text{since } e^{ikx} + e^{-ikx} = 2\cos kx\right]$$

Imagine adding together cosine functions of every freq.
What point on the x axis would you be gaurenteed constructive interference?

Consistent with the wave packet uncertainty principle?

Example from Research: The researchers measured the conductance (g) of a small structure. It looks like g is noisey when magnetic field is increased. Fourier analysis (FT) shows g fluctuates periodically with H.

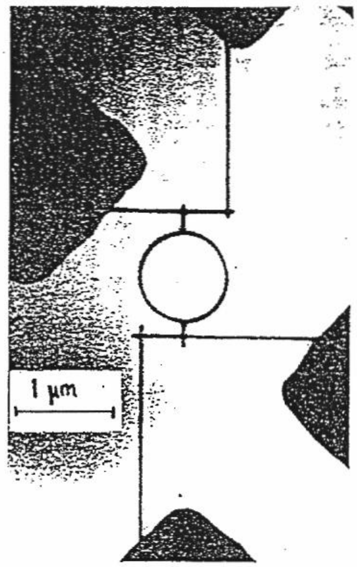


Figure 5.3. A gold ring with four leads (two at the top and left, two at the bottom and right) to study Aharonov-Bohm effects in the quantum diffusive regime.

very weak *h*/2*e* feature is seen. From more detailed experiments (at large magnetic fields) it became clear that this is a harmonic of the AB oscillation and not an AAS contribution, i.e. not due to time-reversed trajectories. From the discussion on the AB and AAS

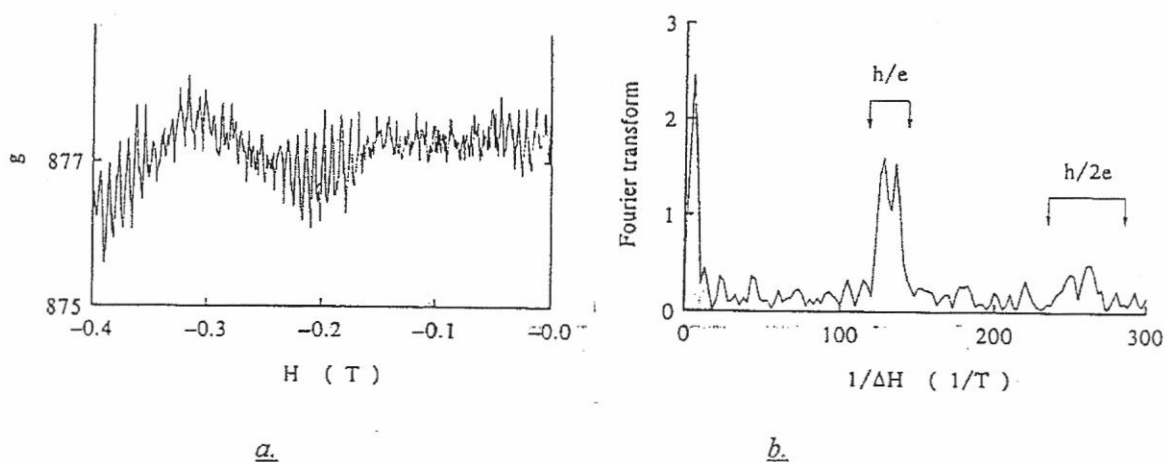


Figure 5.4. Conductance oscillations in the ring of fig. 5.3, clearly demonstrating the AB effect. a: A sample of the data; b: The Fourier transform of the data of a.