

# PART I: CLASSICAL (+ a little quantum theory about phonons)

## Coupled oscillators:

- ① Chain of atoms coupled by springs  
(= chain of frictionless carts coupled by springs)
- ② Chain of pendulums connected by springs
- ③ Chain of LC resonator circuits wired together.

- CONCEPTS:
- Natural freqs
  - Normal modes
  - When coupling is harmonic and only to nearest neighbors we describe ~~normal~~ normal modes using wave vectors.  $\rightarrow$  dispersion relations.
  - First Brillouin Zone
  - Fixed-end vs. periodic B.C.s
  - Quantized energy in a normal mode  $\leftrightarrow$  phonon.
  - Phonon occupation number (Bose-Einstein Statistics)
  - total energy stored in vibrational modes  $\leftrightarrow$  heat capacity
  - Dispersion relation for diatomic lattice.

- EXAMPLE QUESTIONS:
- Write down and solve eqns of motion for a system of coupled oscillators.
  - Use boundary conditions ~~to~~ & dispersion relation to calculate natural freqs.

## PART II : QUANTUM

Theme: LCAO states are a good approximation to the actual electron eigenstates that exist in a lattice of quantum wells.

- CONCEPTS :
- Use a single quantum well to calculate the "atomic orbital"
  - Basis states = The atomic orbital at each lattice site
  - Write the Hamiltonian in the LCAO basis (matrix elements often called  $\alpha$  &  $\beta$ )
  - Write the eigenstates in terms of the LCAO basis.  
One value of  $k$  for each eigenstate.
  - Fill the eigenstates with non-interacting  $e^-$ s (spin up & spin down)
  - Partially-filled band = metal  
Filled band = semiconductor or insulator.
  - Use dispersion relation to understand bandgap and density of states.
  - Light absorption by electronic transitions requires a filled state & empty state separated by  $E_{\text{photon}}$ .

## EXAMPLE QUESTIONS:

- Write out  $\Psi(x,t)$  for an LCAO eigenstate
- Sketch  $\text{Re}\{\Psi(x)\}$  at  $t=0$  for an LCAO eigenstate.
- Find allowed  $k$  given Boundary conditions.
- List eigenenergies of a 5-well system in terms of  $\alpha$  &  $\beta$ .
- Calculate  $g(E)$  from a dispersion relation

# EXAMPLE QUESTIONS FOR FINAL:

## True/False Questions

T F The typical speed of sound in a solid crystal is  $300 \text{ ms}^{-1}$

T F The heat capacity of a diamond at room temp is much higher than the heat capacity of the same diamond at 10K.

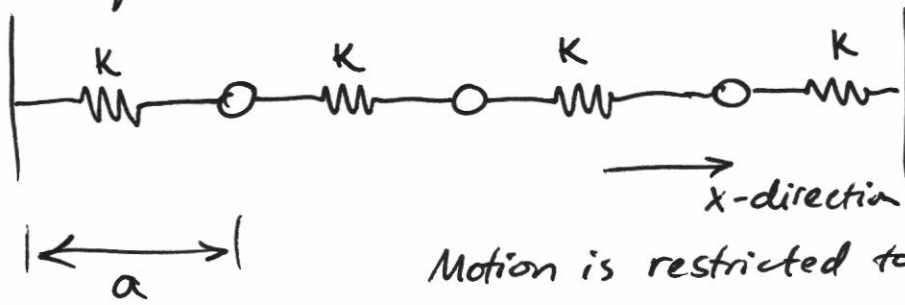
## Short answer questions

- What is the ratio of  $\omega_{\text{Debye}}$  for crystals made of different isotopes: Compare a crystal of  $^{202}\text{Hg}$  to  $^{198}\text{Hg}$ .

Note: Isotope means that proton number is identical, but neutron number is different.

- At least one short answer question will be identical to a quiz question

Long answer question



For normal modes

$$x_1(t) = c_1 A \sin \omega t$$

$$x_2(t) = c_2 A \sin \omega t$$

$$x_3(t) = c_3 A \sin \omega t$$

a) For each normal mode, find an appropriate set of coefficients  $\{c_1, c_2, c_3\}$

b) Find the natural freqs in terms of  $K$  &  $m$ .

- Whiteboard exercises we did in class are good material for exam questions

- Simplified versions of HW problems are good material for exam questions