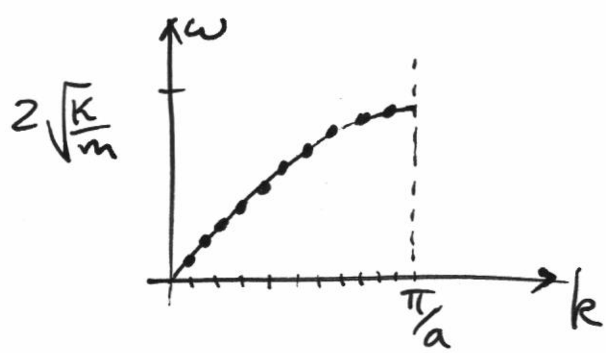
Quiz: Using $\omega = 2\sqrt{\frac{K}{m}} \sin \frac{ka}{2}$

find the 3^{rd} highest natural freq for a system of 5 masses coupled by springs (to each other and to two walls).

Last time

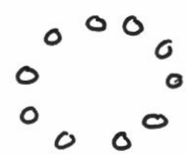
Fixed-end B.C.s

10 masses

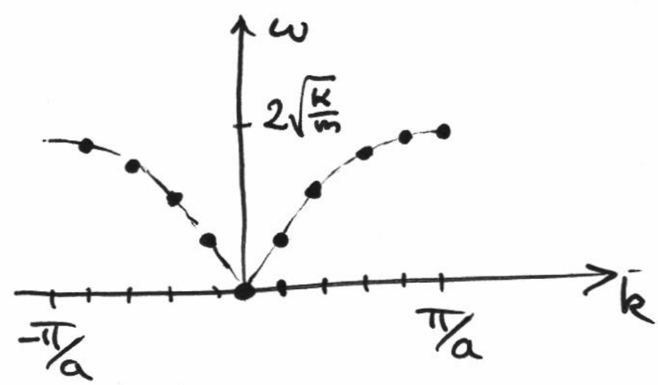


Standing waves.

Periodic B.C.s



10 masses



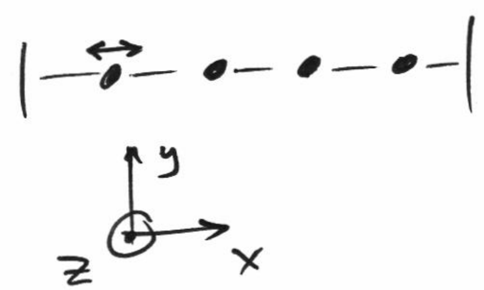
$$\omega = 2\sqrt{\frac{K}{m}} \left| \sin \frac{ka}{2} \right|$$

Traveling waves.

Periodic BCs are the most common choice in Solid state physics. Somewhat arbitrary choice. However, key advantage:

* we can make wave packets out of traveling waves and then discuss wave-particle duality. i.e. Treat electrons as both particles and waves.

OSCILLATORS IN MORE THAN ONE DIMENSION



N = 4

If motion is restricted to x-axis there are 4 normal modes.

If motion is restricted to the x-y plane there are 8 normal modes (2N).

If motion is 3d. There are 12 normal modes (3N).

White board question

How many normal modes are supported by a 1 carat diamond?

THERMAL VIBATIONS IN MATERIALS

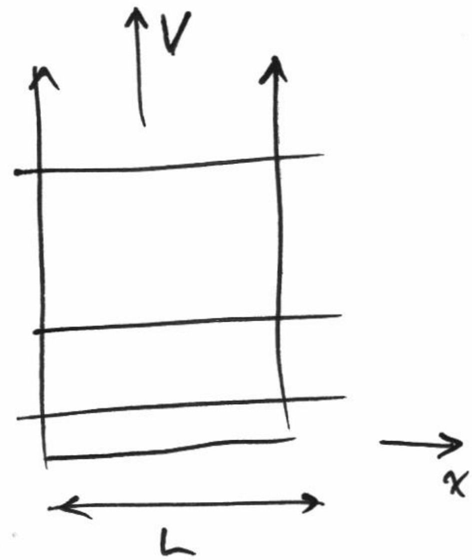
Demonstrate on the computer simulation.

How can I simulate "turning up T" ← temperature
from $T = 0$ to $T > 0$.

CLASSICAL PHYSICS	QUANTUM PHYSICS
Every mode stores energy $k_B T$	Energy in each mode must be a discrete number of "energy quanta"
$$U_{Tot} = 3 N k_B T$$	At low temperature $k_B T <$ "one energy quanta"
(Equipartition theorem applied to systems in thermal eq^b)	→ Modes get "frozen out".
	i.e. Some modes don't store vibrational energy at low T.

THE ENERGY QUANTA FOR VIBRATIONAL MODES

Recall the infinite potential well



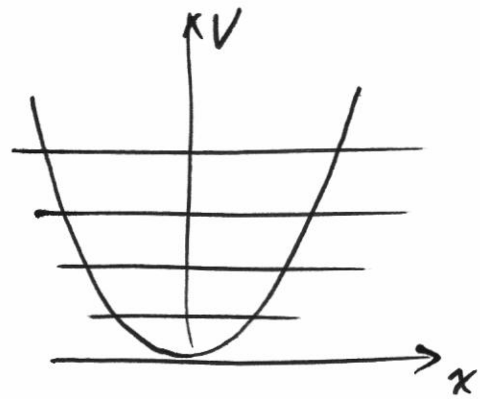
Energy levels are

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

n is the quantum number for the state.

For a mass on a spring $V(x) = \frac{1}{2} k x^2$

A quantum harmonic oscillator has energy levels



$$E_n = \hbar \sqrt{\frac{k}{m}} \left(n + \tfrac{1}{2} \right)$$

quantum number

When many masses move together in a normal mode, k, the energy in that normal mode is

$$E_{k,n} = \hbar \omega_k \left(n_k + \tfrac{1}{2} \right)$$

each normal mode ~~has~~ can have a different quantum no.

n_k is more commonly referred to as "the number of phonons" in a particular mode.

Quantum numbers have to be integer.

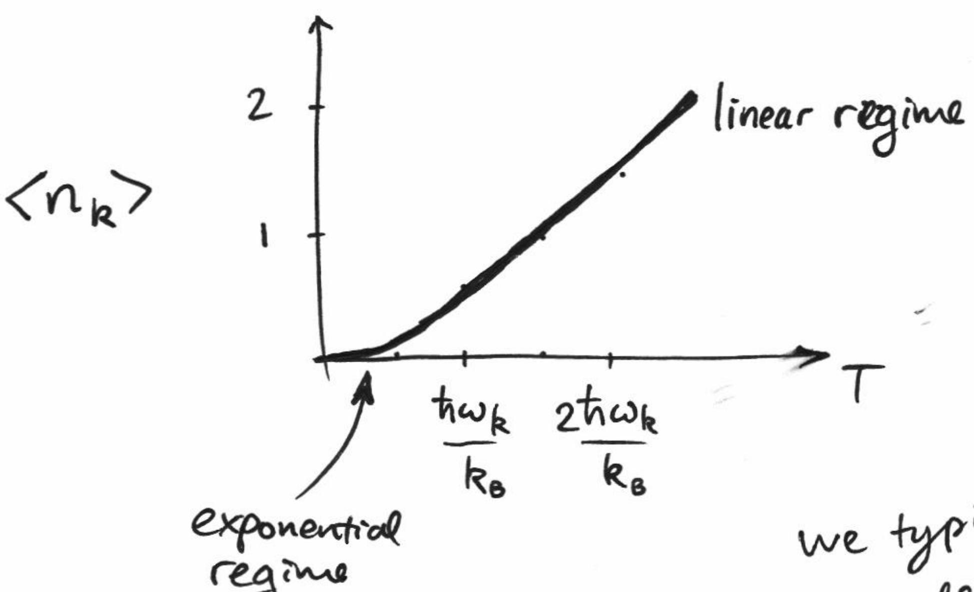
But n_k can fluctuate as a function of time.

The time averaged value of n_k, $\langle n_k \rangle$, is not restricted to integer values.

In thermal eqb

$$\boxed{\langle n_k \rangle = \frac{1}{e^{\hbar\omega_k/k_B T} - 1}}$$

Bose-Einstein Statistics.



we typically say a mode is "frozen out" when $T < \frac{\hbar\omega_k}{k_B}$