Quantum Calculations on a Ring I

Before you begin, recall that an arbitrary state $|\Phi\rangle$ can be written in the L_z eigenbasis as

$$|\Phi\rangle \doteq \begin{pmatrix} \vdots \\ \langle 2|\Phi\rangle \\ \langle 1|\Phi\rangle \\ \langle 0|\Phi\rangle \\ \langle -1|\Phi\rangle \\ \langle -2|\Phi\rangle \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ a_2 \\ a_1 \\ a_0 \\ a_{-1} \\ a_{-2} \\ \vdots \end{pmatrix}$$

In this activity, your group will carry out calculaions on each of the following normalized abstract quantum states on a ring:

$$|\Phi_a\rangle = \sqrt{\frac{2}{12}}\,|3\rangle + \sqrt{\frac{1}{12}}\,|1\rangle + \sqrt{\frac{3}{12}}\,|0\rangle + \sqrt{\frac{2}{12}}\,|{-1}\rangle + \sqrt{\frac{1}{12}}\,|{-3}\rangle + \sqrt{\frac{3}{12}}\,|{-4}\rangle$$

$$|\Phi_b\rangle \doteq \begin{pmatrix} \vdots \\ 0 \\ \sqrt{\frac{2}{12}} \\ 0 \\ \sqrt{\frac{1}{12}} \\ \sqrt{\frac{3}{12}} \\ \sqrt{\frac{2}{12}} \\ 0 \\ \sqrt{\frac{1}{12}} \\ \sqrt{\frac{3}{12}} \\ \vdots \end{pmatrix}$$

$$\Phi_c(\phi) = \sqrt{\frac{1}{24\pi}}\left(\sqrt{2}\left(e^{i3\phi} + e^{-i\phi}\right) + \left(e^{i\phi} + e^{-i3\phi}\right) + \sqrt{3}(1 + e^{-i4\phi})\right)$$

For each question state the postulate(s) of quantum mechanics you use to complete the calculation and show explicitly how you use that postulate to answer the question.

1) If you measured the z-component of angular momentum for each state, what is the probability that you would obtain:

Group 1: $\hbar$	Group 2: $-4\hbar$	Group 3: $-3\hbar$	Group 4: $4\hbar$
Group 5: $-\hbar$	Group 6: 0	Group 7: $-2\hbar$	Group 8: $3\hbar$

2) If you measured the energy for each state, what is the probability that you would obtain:

Group 1: $\frac{9\hbar^2}{2I}$ Group 2: 0 Group 3: $\frac{4\hbar^2}{2I}$ Group 4: $\frac{\hbar^2}{2I}$

Group 5: $\frac{16\hbar^2}{2I}$ Group 6: $\frac{-\hbar^2}{2I}$ Group 7: $\frac{9\hbar^2}{2I}$ Group 8: $\frac{\hbar^2}{2I}$

3) How are the calculations you made for the different state representations similar and different? Be prepared to compare and contrast the calculations you made for each of the different representations (ket, matrix, wavefunction).

by Corinne Manogue and Kerry Browne

Quantum Calculations on a Ring II

In this activity, your group will carry out calculations on the following normalized abstract quantum state on a ring:

$$|\Psi\rangle = \sqrt{\frac{1}{4}}\left(\sqrt{2}\,|1\rangle + |{-3}\rangle + |3\rangle\right)$$

1) You carry out a measurement to determine the energy of the particle at time t=0. Calculate the probability that you measure the energy to be $\frac{9\hbar^2}{2I}$. What representation/basis did you use to do this calculation and why did you use this representation?

2) You carry out a measurement on the location of the particle at time, t=0. Calculate the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{3}$. What representation/basis did you use to do this calculation and why did you use this representation?

3) You carry out a measurement to determine the energy of the particle at time $t = \frac{2I}{\hbar}\frac{\pi}{16}$. Calculate the probability that you measure the energy to be $\frac{9\hbar^2}{2I}$. What representation/basis did you use to do this calculation and why did you use this representation?

4) You carry out a measurement on the location of the particle at time, $t = \frac{2I}{\hbar}\frac{\pi}{16}$. Calculate the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{3}$. What representation/basis did you use to do this calculation and why did you use this representation?

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Quantum Calculations on a Ring III

Consider the following normalized abstract quantum state on a ring:

$$\Phi(\phi) = \sqrt{\frac{8}{5\pi r_0}} \cos^3(2\phi)$$

1) If you measured the z-component of angular momentum, what is the probability that you would obtain $2\hbar$? $-3\hbar$?

2) If you measured the z-component of angular momentum, what other possible values could you obtain with non-zero probability?

3) If you measured the energy, what possible values could you obtain with non-zero probability?

4) What is the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{2}$?

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