DAY 8

PH424 PART II !

Now following McIntyre, starting with Chpt 5.

Schrödinger's Eqn

- Another example of a partial differential eqn that governs a wavefn $\Psi(x,y,z,t)$

- Allows us to explain the structure of atoms, molecules ... the microscopic world.

- Even the behavior of an isolated free electron defies ~~the~~ explaination of classical physics

→ see YouTube video link.

Review material from spins paradigm

You asked an electron

"Are you spin up or spin down?"

Now we will ask

"Where are you?"

Spin is a discrete variable

Position is a continuous variable

How will the mathematical descriptions relate to one another?

If $|\psi\rangle$ is an arbitrary state vector

Probability of spin up measurement is $|\langle + | \psi \rangle|^2$

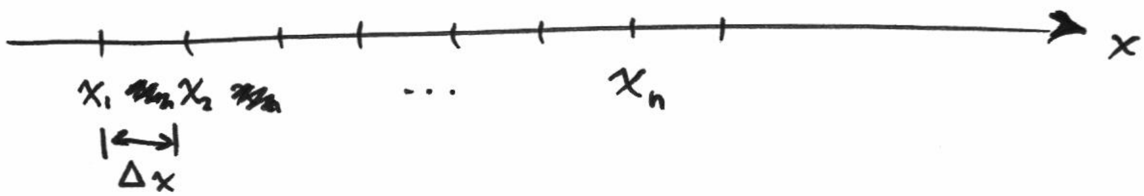
We can equally well write $|\psi\rangle$ as a vector where

$$|+\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |-\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and $|\psi\rangle = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

Probability of a spin up measurement is $|c_1|^2$.

What if I want to calculate the probability of measuring the x position of an electron to be at a certainty segment of the x axis.



$$|x_1\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

Basis ~~state~~ vector for ~~with~~ position information, electron between $x_1 \longleftrightarrow x_1 + \Delta x$

$$|x_2\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

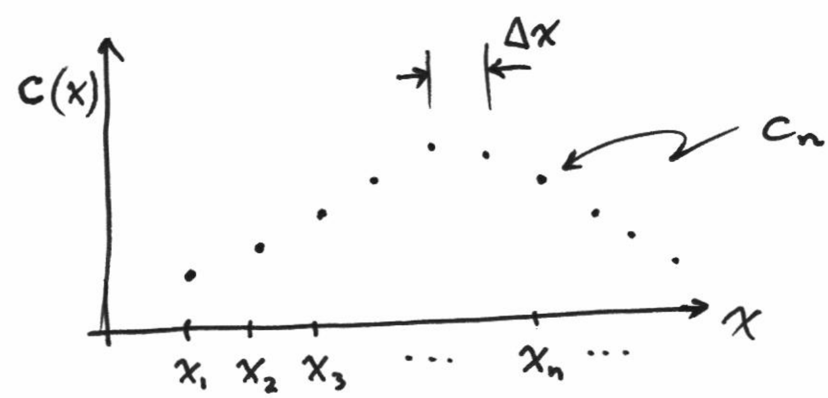
Basis ~~state~~ vector for position, electron between $x_2 \longleftrightarrow x_2 + \Delta x$

etc.

Arbitrary state vector is ~~then~~ (in position basis)

$$|\psi\rangle = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \\ \vdots \end{bmatrix}$$

Represented graphically



$|c_n|^2$ is probability of ~~finding~~ measuring the e^- between $x_n \leftrightarrow x_n + \Delta x$

(Note $\sum_n |c_n|^2 = 1$)

Let Δx become very very small $\longrightarrow dx$

c(x) becomes a continuous fn that we will call $\psi(x)$.

$\psi(x)$ is normalized just like the c_n coefficients

$$\sum_n |c_n|^2 = 1 \qquad \text{for finite } \Delta x$$

$\vdots$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1 \qquad \text{in the limit of infinitessimal } dx.$$

The Quantum Mechanical wave function.

This is the first step in building the mathematical machinary to ask "where is the electron?".

Now, how do we calculate $\psi(x,t)$?

It must satisfy the following Partial Differential Eq^{n}:

$$\left(\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right) \psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t)$$

This is the Schrödinger Eqⁿ written in terms of "DIFFERENTIAL OPERATORS" rather than MATRIX OPERATORS.

Worksheet: Getting used to differential operators.