

DAY 7

Last time

① PH 424

$$W_{K.E.} = \frac{1}{2} \mu \left( \frac{\partial \psi}{\partial t} \right)^2$$

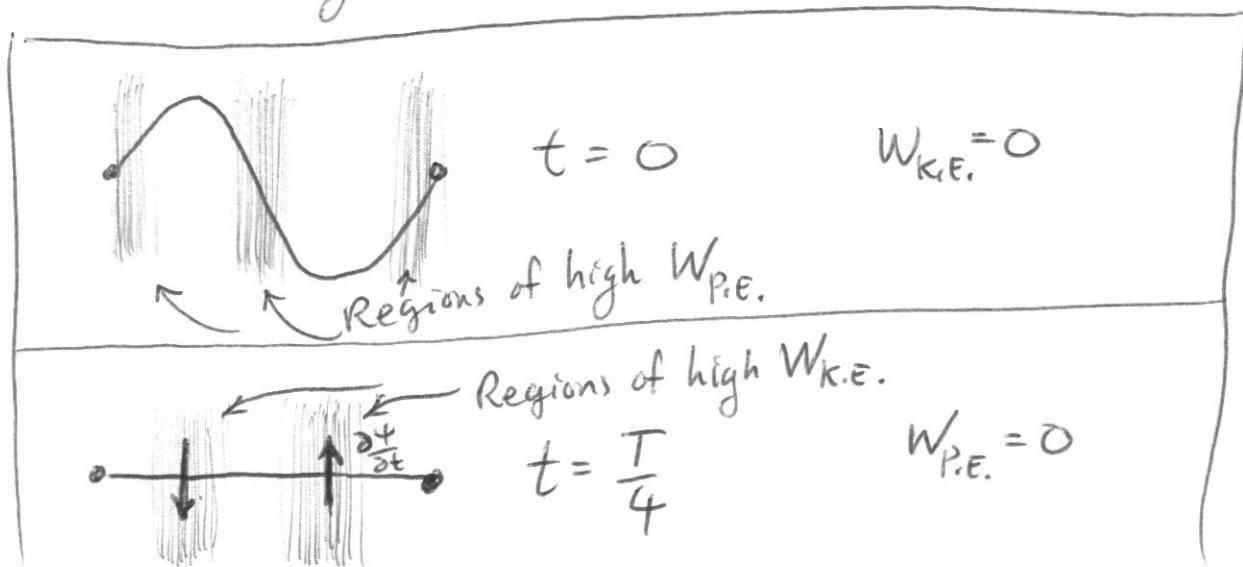
$$W_{P.E.} = \frac{1}{2} \tau \left( \frac{\partial \psi}{\partial x} \right)^2$$

For a traveling wave



These regions move to the right at  $v_{phase}$ .  
Energy is transported to the right.

For a standing wave

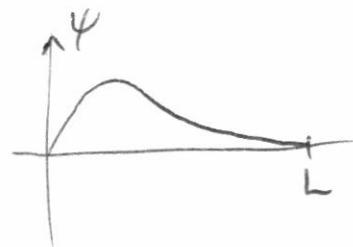


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# SUPERPOSITION OF TWO STANDING WAVES (worksheet).

Stretched string starts in this shape:

$$\psi(x, t=0) = A \sin\left(\frac{\pi x}{L}\right) \left(1 + \cos\left(\frac{\pi x}{L}\right)\right)$$



find  $\psi(x, t)$ .

I want to express this ~~in terms~~<sup>as a sum</sup> of simple sine and cosine functions. ~~with diff~~

$$= A \sin \frac{\pi x}{L} + \frac{A}{2} \sin \frac{2\pi x}{L} \quad [\text{using Trig identity}]$$

$\uparrow$                                      $\uparrow$   
 $k = \frac{\pi}{L}$                              $k = \frac{2\pi}{L}$

Recall the general solution to the <sup>non-dispersive</sup> wave eq'n

$$\psi(x, t) = \sum_i \left\{ A_i \cos k_i x \cos \omega_i t + B_i \cos k_i x \sin \omega_i t + C_i \sin k_i x \cos \omega_i t + D_i \sin k_i x \sin \omega_i t \right\}$$

where  $\frac{\omega_i}{k_i} = v$  for every  $i$ .

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We are told that  $\left. \frac{\partial \psi}{\partial t} \right|_{t=0} = 0$ .

Therefore, all time dependence must be of the form  $\cos \omega_i t$ .

$$\Rightarrow \boxed{\psi(x,t) = A \sin \frac{\pi x}{L} \cos \frac{\pi v t}{L} + \frac{A}{2} \sin \frac{2\pi x}{L} \cos \frac{2\pi v t}{L}}$$

Note the similarity to time dependent state vectors in quantum mechanics

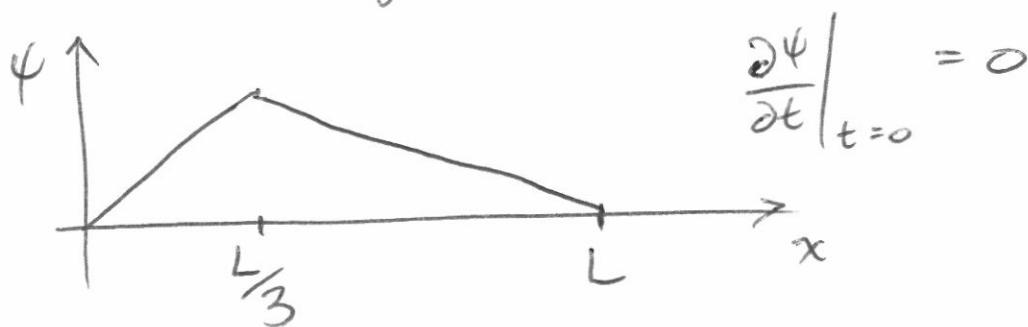
$$* \quad \frac{1}{\sqrt{2}} e^{-i \frac{E_+}{\hbar} t} |+\rangle + \frac{1}{\sqrt{2}} e^{-i \frac{E_-}{\hbar} t} |-\rangle$$

↑

different basis states  
have different time dependence.

Introduction to Friday's HW question

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Need to

Project this function onto standing waves  
that have different values of  $k$ .