

DAY 7

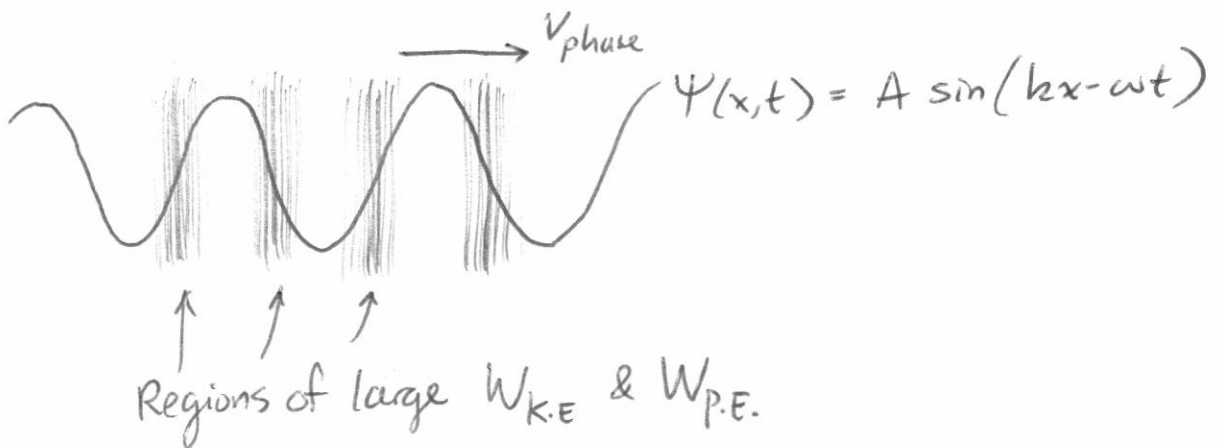
Last time

PH 424

$$W_{k.e.} = \frac{1}{2} \mu \left( \frac{\partial \psi}{\partial t} \right)^2$$

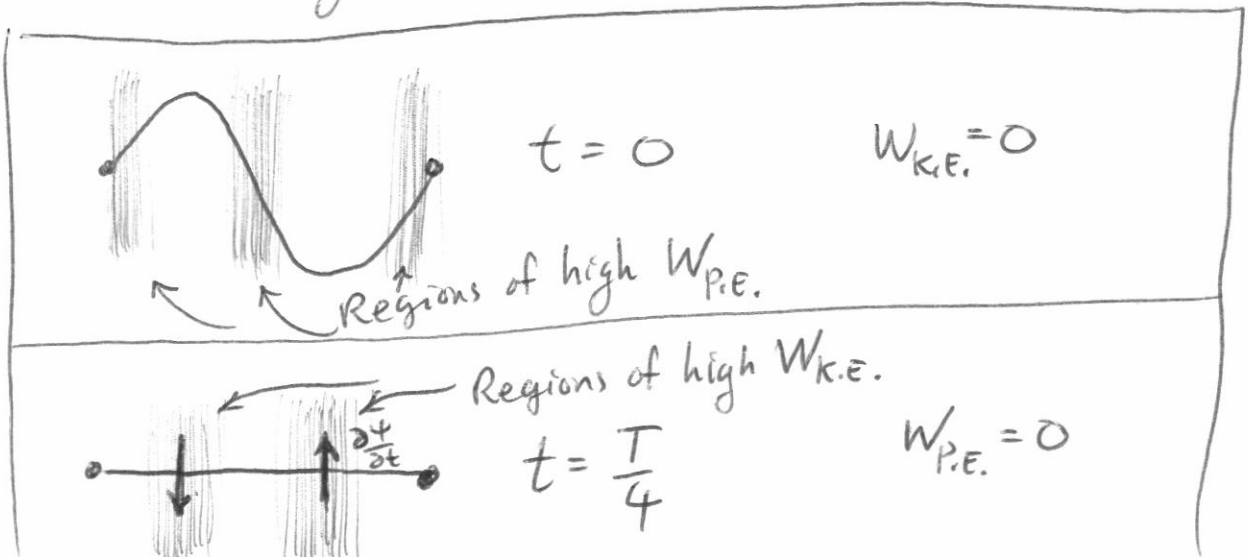
$$W_{p.e.} = \frac{1}{2} \tau \left( \frac{\partial \psi}{\partial x} \right)^2$$

For a traveling wave



These regions move to the right at  $v_{phase}$ .  
Energy is transported to the right.

For a standing wave

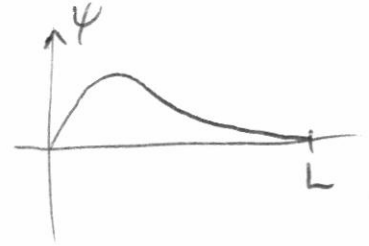


(2)

# SUPERPOSITION OF TWO STANDING WAVES (worksheets).

Stretched string starts in this shape:

$$\Psi(x, t=0) = A \sin\left(\frac{\pi x}{L}\right) \left(1 + \cos\left(\frac{\pi x}{L}\right)\right)$$



find  $\Psi(x, t)$ .

I want to express this ~~in terms~~ <sup>as a sum</sup> of simple sine and cosine functions. ~~with dif~~

$$= A \sin \frac{\pi x}{L} + \frac{A}{2} \sin \frac{2\pi x}{L}$$

[using Trig identity]

$$\uparrow$$

$$k = \frac{\pi}{L}$$

$$\uparrow$$

$$k = \frac{2\pi}{L}$$

Recall the general solution to the <sup>non-dispersive</sup> wave eq<sup>n</sup>

$$\Psi(x, t) = \sum_i \left\{ \begin{array}{l} A_i \cos k_i x \cos \omega_i t \\ + B_i \cos k_i x \sin \omega_i t \\ + C_i \sin k_i x \cos \omega_i t \\ + D_i \sin k_i x \sin \omega_i t \end{array} \right\}$$

where  $\frac{\omega_i}{k_i} = v$  for every  $i$ .

③

We are told that  $\left. \frac{\partial \psi}{\partial t} \right|_{t=0} = 0$ .

Therefore, all time dependence must be of the form  $\cos \omega_i t$ .

$$\Rightarrow \boxed{\psi(x,t) = A \sin \frac{\pi x}{L} \cos \frac{\pi v}{L} t + \frac{A}{2} \sin \frac{2\pi x}{L} \cos \frac{2\pi v}{L} t}$$

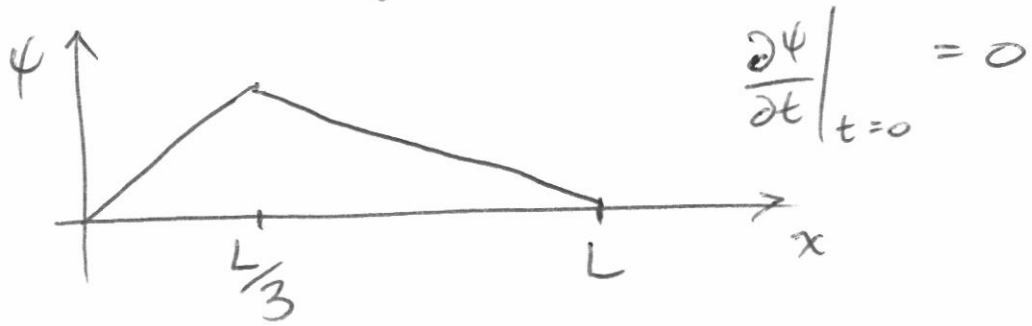
Note the similarity to time dependent state vectors in quantum mechanics

$$\psi = \frac{1}{\sqrt{2}} e^{-i \frac{E_+}{\hbar} t} |+\rangle + \frac{1}{\sqrt{2}} e^{-i \frac{E_-}{\hbar} t} |-\rangle$$

↑  
different basis states  
have different time dependence.

(4)

Introduction to Friday's HW question



~~Need to~~

Project this function onto standing waves that have different values of  $k$ .