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DAY 6

Ph 424

Transverse waves

- Displacement of a guitar string.

(The displacement is perpendicular to the direction of the phase velocity)

Longitudinal Waves.

- Sound wave.
- Electron displacement in a coax cable.

(The displacement is "along" the direction of the phase velocity)

Tell me something you know about energy ~~energy~~ in the harmonic oscillator example

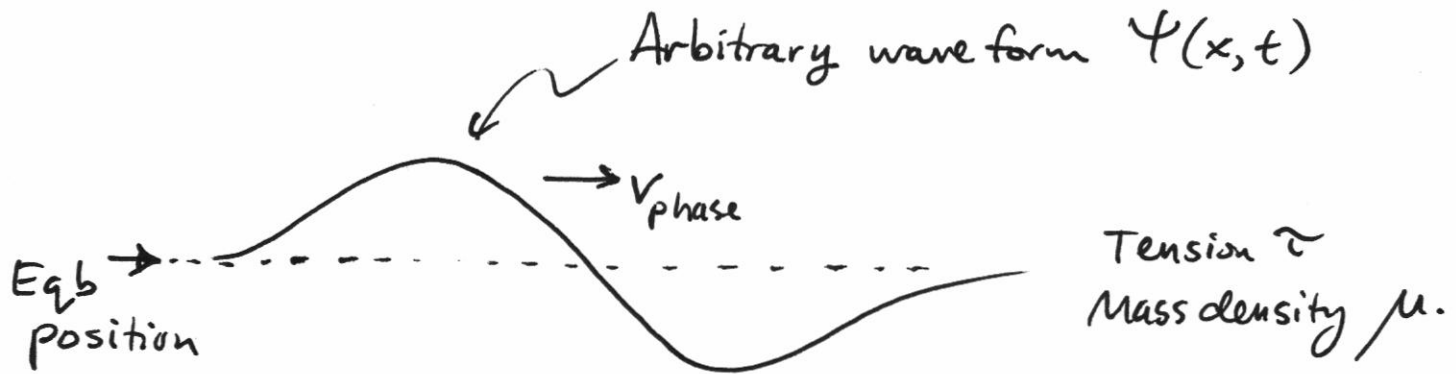
K.E. + P.E. = Total Energy

$\frac{1}{2}mv^2$  (with an arrow pointing up and right towards the equation above)

$\frac{1}{2}kx^2$  (with an arrow pointing up and left towards the equation above)

(2)

For a <sup>transverse</sup> wave on a string under tension,  $\tau$ , we'll need to define energy density,  $\frac{\text{Energy}}{\text{Length}}$ .



Find energy density ~~as a function~~ in terms of  $\Psi(x, t)$ , tension  $\tau$  and mass density  $\mu.$

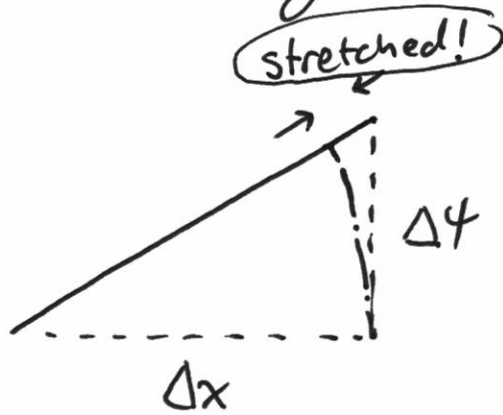
The up/down velocity of a short segment is  $\frac{\partial \Psi}{\partial t}$ .

$$\text{K.E.} = \frac{1}{2} m \left( \frac{\partial \Psi}{\partial t} \right)^2 = \frac{1}{2} \mu \Delta x \left( \frac{\partial \Psi}{\partial t} \right)^2$$

$$W_{\text{K.E.}} = \frac{\text{K.E.}}{\Delta x} = \frac{1}{2} \mu \left( \frac{\partial \Psi}{\partial t} \right)^2 \quad \left[ \text{Kinetic energy per length} \right]$$

(3)

The ~~end to end distance~~ <sup>contour length</sup> of the string increases when the wave ~~is~~ is present. Consider a place on the string where slope is  $\frac{\Delta\psi}{\Delta x}$



The distance that the string has been stretched is

$$\text{stretch length} = \sqrt{(\Delta x)^2 + (\Delta \psi)^2} - \Delta x$$

$$\approx \Delta x \left( 1 + \left( \frac{\Delta \psi}{\Delta x} \right)^2 \right)^{1/2} - \Delta x$$

$$\approx \Delta x \left( 1 + \frac{1}{2} \left( \frac{\partial \psi}{\partial x} \right)^2 \right) - \Delta x$$

Taylor expansion.

$$= \frac{1}{2} \Delta x \left( \frac{\partial \psi}{\partial x} \right)^2$$

(4)

Potential energy stored in this segment of string is equal to the work done stretching the string

$$P.E. = \text{Work Done} = (\text{Force}) \cdot (\text{Stretch Length})$$

$$= \frac{1}{2} \tau \Delta x \left( \frac{\partial \psi}{\partial x} \right)^2$$

$$W_{P.E.} = \frac{1}{2} \tau \left( \frac{\partial \psi}{\partial x} \right)^2$$

[Potential Energy per Length]