DAY 6

Ph 424

Transverse waves

- Displacement of a guitar string.

(The displacement is perpendicular to the direction of the phase velocity)

Longitudinal Waves.

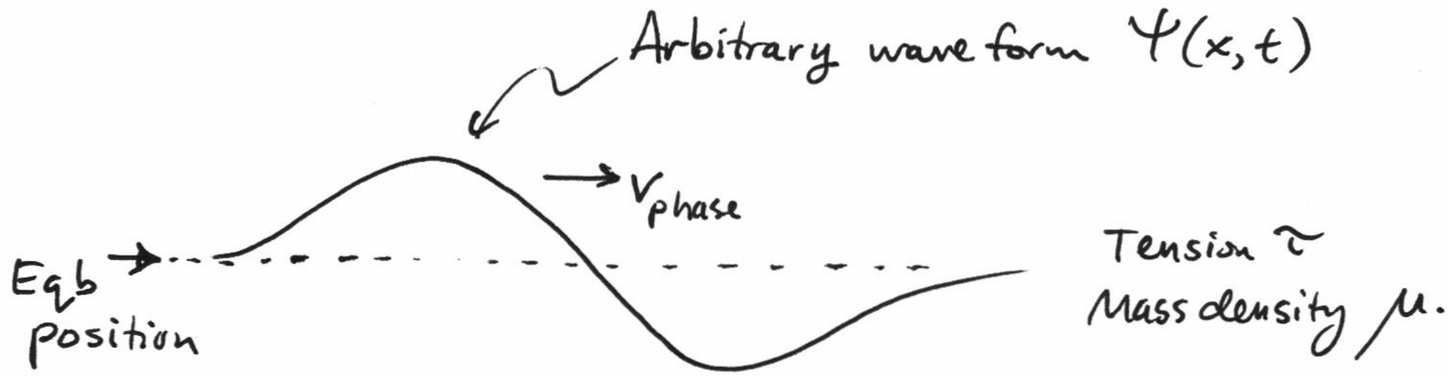
- Sound wave.
- Electron displacement in a coax cable.

(The displacement is "along" the direction of the phase velocity)

Tell me something you know about energy in the harmonic oscillator example

$$\underbrace{K.E.}_{\frac{1}{2}mv^2} + \underbrace{P.E.}_{\frac{1}{2}kx^2} = \text{Total Energy}$$

For a transverse wave on a string under tension, τ, we'll need to define energy density, $\dfrac{\text{Energy}}{\text{Length}}$.



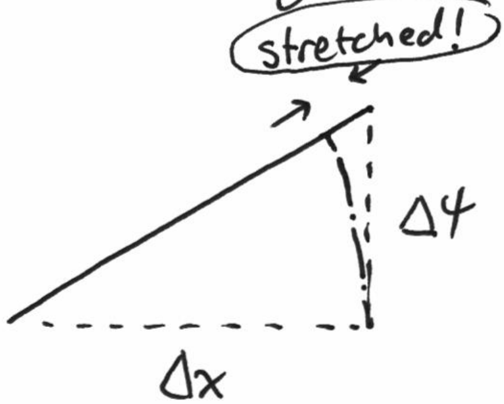
Find energy density ~~as a fun~~ in terms of $\Psi(x,t)$, tension τ and mass density μ.

The up/down velocity of a short segment is $\dfrac{\partial \Psi}{\partial t}$.

$$K.E. = \frac{1}{2} m \left(\frac{\partial \Psi}{\partial t}\right)^2 = \frac{1}{2} \mu \Delta x \left(\frac{\partial \Psi}{\partial t}\right)^2$$

$$W_{K.E.} = \frac{K.E.}{\Delta x} = \frac{1}{2} \mu \left(\frac{\partial \Psi}{\partial t}\right)^2 \qquad \text{[Kinetic energy per length]}$$

The ~~end to end distance~~ contour length of the string increases when the wave is present. Consider a place on the string where slope is $\frac{\Delta \psi}{\Delta x}$



The distance that the string has been stretched is

$$\text{stretch length} = \sqrt{(\Delta x)^2 + (\Delta \psi)^2} - \Delta x$$

$$= \Delta x \left(1 + \left(\frac{\Delta \psi}{\Delta x} \right)^2 \right)^{1/2} - \Delta x$$

$$\cong \Delta x \left(1 + \tfrac{1}{2} \left(\frac{\partial \psi}{\partial x} \right)^2 \right) - \Delta x \qquad \text{Taylor expansion.}$$

$$= \tfrac{1}{2} \Delta x \left(\frac{\partial \psi}{\partial x} \right)^2$$

Potential energy stored in this segment of string is equal to the work done stretching the string

$$P.E. = \text{Work Done} = (\text{Force})\cdot(\text{stretch Length})$$

$$= \frac{1}{2}\tau\,\Delta x\left(\frac{\partial\psi}{\partial x}\right)^2$$

$$W_{P.E.} = \frac{1}{2}\tau\left(\frac{\partial\psi}{\partial x}\right)^2 \qquad \left[\text{Potential Energy per Length}\right]$$