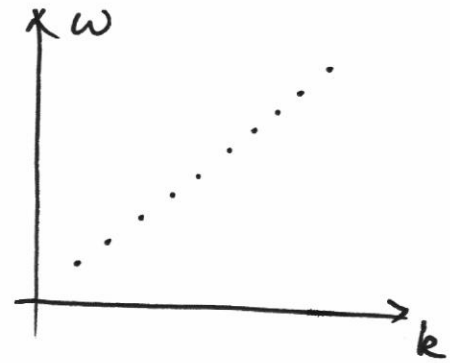
Last time



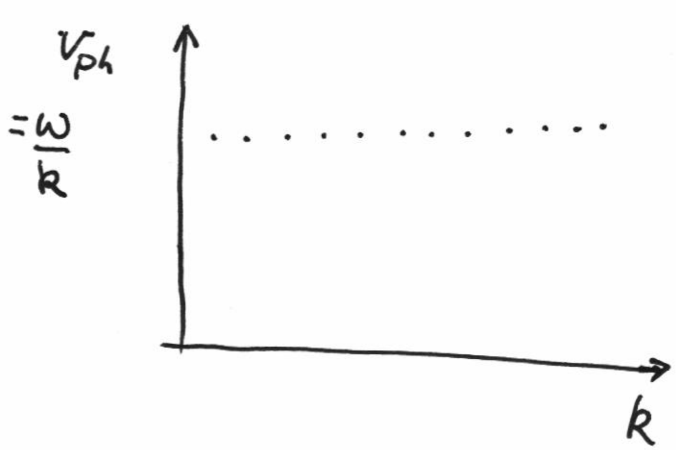
Frequency Dispersion relation for string under tension.

Each dot represents a different solution to the wave eqn

$$\psi_1(x,t) = A \sin k_1 x \sin \omega_1 t$$
$$\psi_2(x,t) = A \sin k_2 x \sin \omega_2 t$$
$$\vdots$$

etc.

For every ω & k pair we find $\boxed{\frac{\omega}{k} = v}$ const.



Phase velocity dispersion relation for string under tension.

"Non-dispersive."

Will it remain non-dispersive for all values of k?

$$k = \frac{2\pi}{\text{string diameter}} \quad ?$$

$$k = \frac{2\pi}{\text{Atomic spacing}} \quad ?$$

Lets look at the phase velocity dispersion relation for water waves

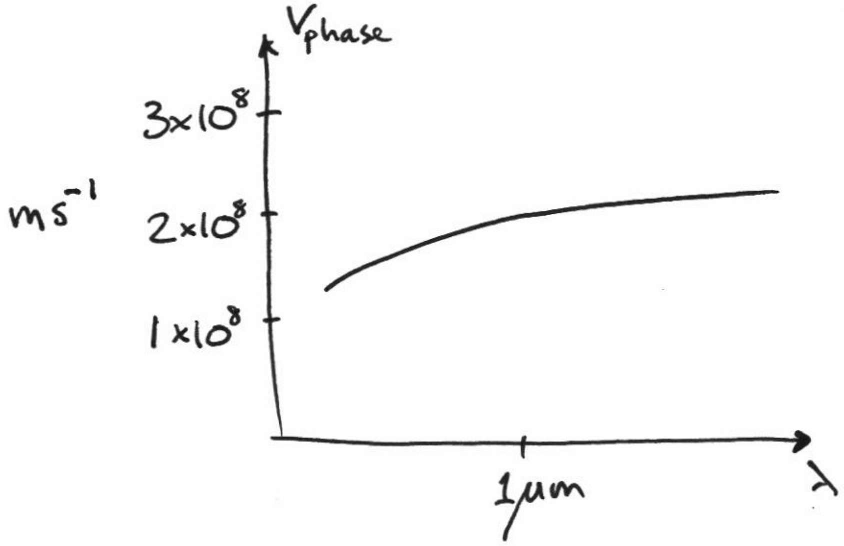
- see website
- see wave machine.

Phase velocity dispersion relation for light waves in glass

- see website.

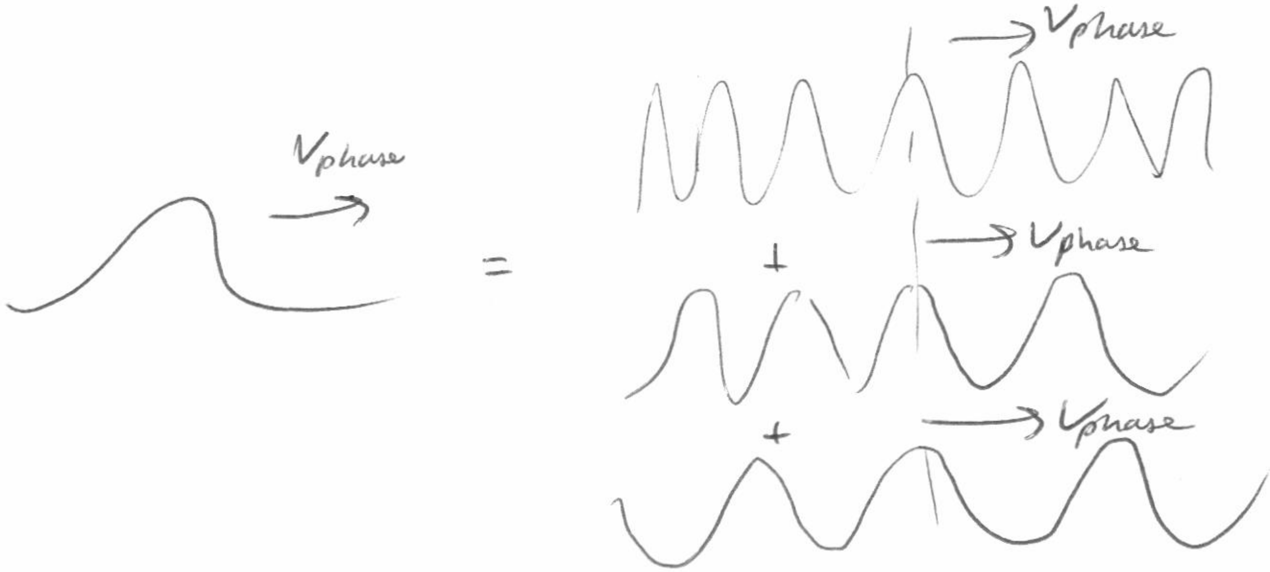
$$\boxed{v_{phase} = \frac{c}{n}}$$

refractive index.



Not flat

Non-dispersive systems are great for sending pulses of information



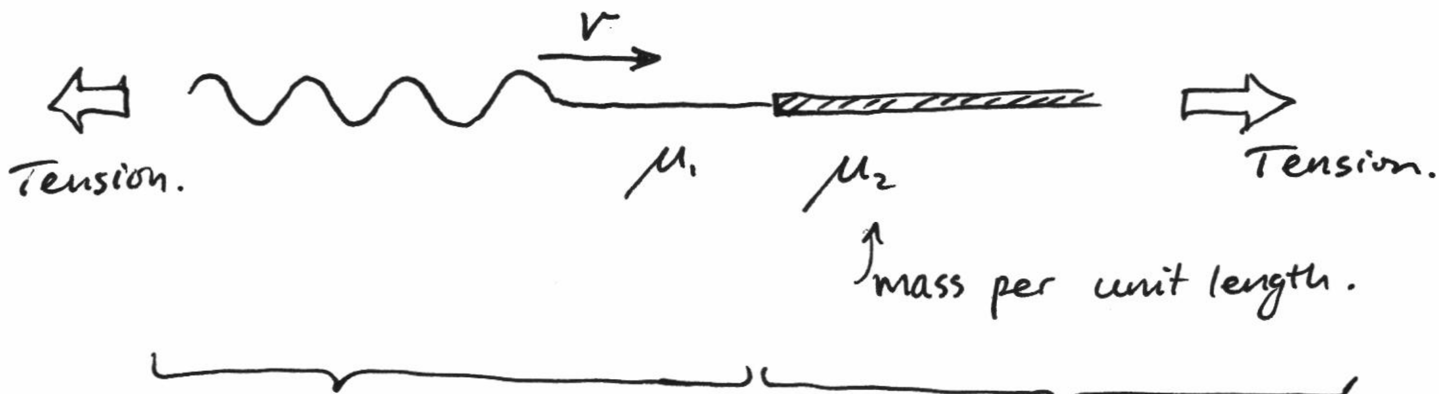
Every Fourier component will move at the same phase velocity.
The pulse shape is perserved.

How about this system?

Fiber optic cable

New topic Reflection/Transmission.

To keep it simple we will only talk about non-dispersive systems.



$$T\frac{\partial^2\psi}{\partial x^2} = \mu_1\frac{\partial^2\psi}{\partial t^2} \qquad\qquad T\frac{\partial^2\psi}{\partial x^2} = \mu_2\frac{\partial^2\psi}{\partial t^2}$$

Left side and right side are governed by different wave eqns. What will happen?

Left

$A\sin(kx-\omega t)$, incident

$B\sin(-kx-\omega t)$, reflected

Right

$C\sin(k'x-\omega t)$, transmitted.

See animation. Note the mixture of standing wave + traveling.

- How would I choose μ_2 to get a pure standing wave?

$\mu_2 \to \infty$, lots of inertia, right rope won't be moved.

- $\mu_2 \to 0$, what happens?

This light weight string wants to be straight, has no interia.

standing wave?

I want to calculate B & C for different pairs of μ_1 & μ_2.

B: reflection

C: transmission

How? Apply B.C.s

Easiest to solve this with complex #s.

$$\psi_{Left} = Re\left[A e^{i(k_1 x - \omega t)} + B e^{i(k_1 x - \omega t)} \right]$$

$$\psi_{Right} = Re\left[C e^{i(k_2 x - \omega t)} \right]$$

We will work with the full complex #s. The imaginary component are coming along for the ride. Won't actually use the Im components.

$$\left.\frac{\partial \psi_{Left}}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_{Right}}{\partial x}\right|_{x=0}$$

$$\left.\left(i k_1 A e^{i(k_1 x - \omega t)} - i k_1 B e^{i(-k_1 x - \omega t)} \right)\right|_{x=0} = i k_2 C e^{i(k_2 x - \omega t)}$$

$$\boxed{k_1 A - k_1 B = k_2 C}$$

$$\psi_L(x=0) = \psi_R(x=0$$

$$\boxed{A + B = C}$$