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DAY 2

PH424

webpage... show class notes from Day 1.

★ Pop Quiz

Last time: Phase velocity $v_{ph} = \frac{\omega}{k} \quad (= \frac{f}{\lambda})$

Traveling waves always contain a $(kx - \omega t)$ term.

Write an expression for a parabola moving to the left.
 $\psi(x,t) = (kx - \omega t)^2$



Demo: Standing wave. (Not all waves ^{fn's} are traveling).

Write an expression describing a standing wave.

$$\sin(kx + \phi_1) \sin(\omega t + \phi_2)$$

Note that the function is separable

$$\psi(x,t) = X(x)T(t)$$

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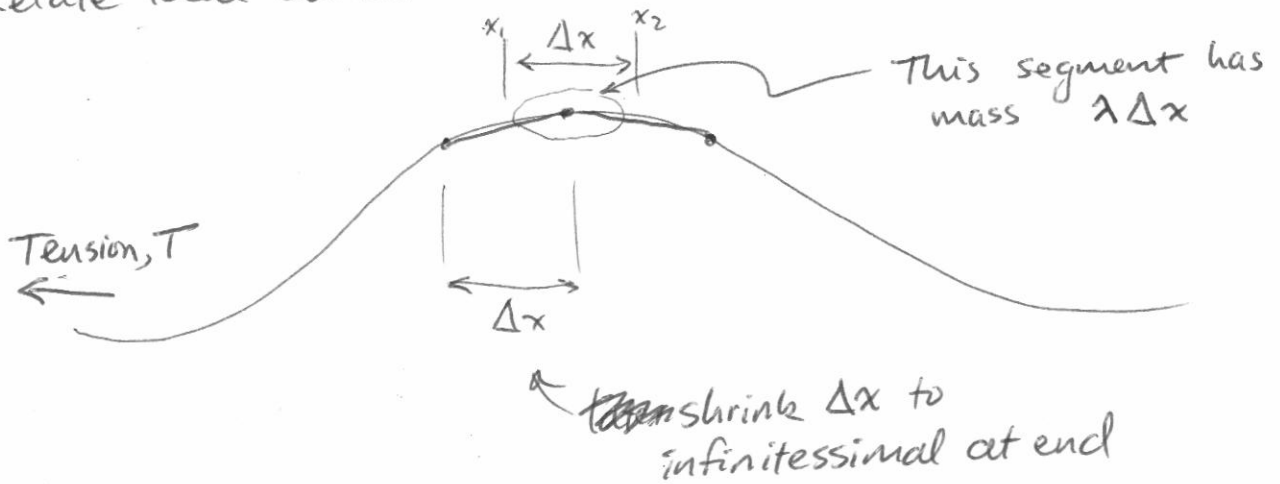
Stretched strings are straight lines (in eqb).

$$\frac{\partial^2 \psi}{\partial x^2} = 0 \quad \text{where } \psi \text{ is displacement from eqb.}$$

If $\frac{\partial^2 \psi}{\partial x^2} \neq 0$ there is a restoring force pushing string back to eqb.

Therefore (by $F=ma$) there is acceleration.

Relate local curvature to local acceleration



Tension is pulling the segment down

$$F_z = T \left(\frac{\partial \psi}{\partial x} \Big|_{x_1} - \frac{\partial \psi}{\partial x} \Big|_{x_2} \right)$$

$$ma = T \frac{\partial^2 \psi}{\partial x^2} \Delta x$$

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$$\frac{\partial^2 \psi(x,t)}{\partial t^2} = \frac{T}{\lambda} \frac{\partial^2 \psi(x,t)}{\partial x^2}$$

Wave eqn for string under tension T .
with linear mass density λ .

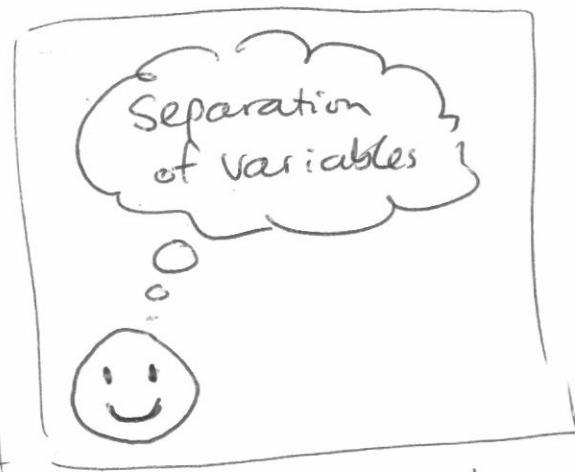
$\psi(x,t) = A \sin(kx - \omega t)$ satisfies * ?

More examples
in HW.

Yes ~~only~~, if $\frac{\omega}{k} = \sqrt{\frac{T}{\lambda}}$, the wave eqn is satisfied.

Guess and check is useful, but how do I find
the most general solution to a 2nd order
linear partial diff eqn?

Introduce a technique that comes up again & again
in physics:



Because if S.o.V. doesn't work, your up the creek w/o a paddle

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Get class to work thru this part

$$\frac{\partial^2}{\partial x^2} \psi(x,t) = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \psi(x,t)$$

$$\psi(x,t) \rightarrow X(x)T(t)$$

$$T(t) \frac{\partial^2}{\partial x^2} X(x) = \frac{1}{v^2} X(x) \frac{\partial^2}{\partial t^2} T(t)$$

$$\frac{1}{X(x)} \frac{\partial^2}{\partial x^2} X(x) = \frac{1}{v^2} \frac{1}{T(t)} \frac{\partial^2}{\partial t^2} T(t)$$

$$\frac{1}{X(x)} \frac{d^2}{dx^2} X(x) = -k^2$$

$$\frac{1}{v^2} \frac{1}{T(t)} \frac{d^2}{dt^2} T(t) = -k^2$$

[choosing a const that has the right dimensions]

Coupled ordinary ^{2nd order} D.E.s ~~Much easier~~

Solutions are now tractable.

Note, ~~k~~ k^2 could be positive or negative, and can take any value.

I'll focus on $k^2 > 0$.

~~the~~ Remember ^{the} general solⁿ of a second order D.E. has two arbitrary constants [^] and two linearly independent functions

If $k^2 > 0$, then

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$$X(x) = A \sin kx + B \cos kx$$

$$T(t) = C \sin \underbrace{kv t}_{=\omega} + D \cos \underbrace{kv t}_{=\omega}$$

$$\psi(x,t) = X(x)T(t) \quad \cancel{A' \cos kx \cos \omega t}$$

$$= A' \sin kx \sin \omega t \quad \cancel{A' \cos kx \sin \omega t}$$

$$+ B' \sin kx \cos \omega t$$

$$+ C' \cos kx \sin \omega t$$

$$+ D' \cos kx \cos \omega t$$

Slinky demo?

where k can have any value

and $\omega = kv$

where v is const from the wave eqn.

? Are these traveling or standing waves?

If this is the general soln, how can we reconcile traveling wave solns like

$$\psi(x,t) = A \sin(kx - \omega t)$$

?

Computer demo

What about arbitrary pulse shapes?

Computer demo

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Trig identity

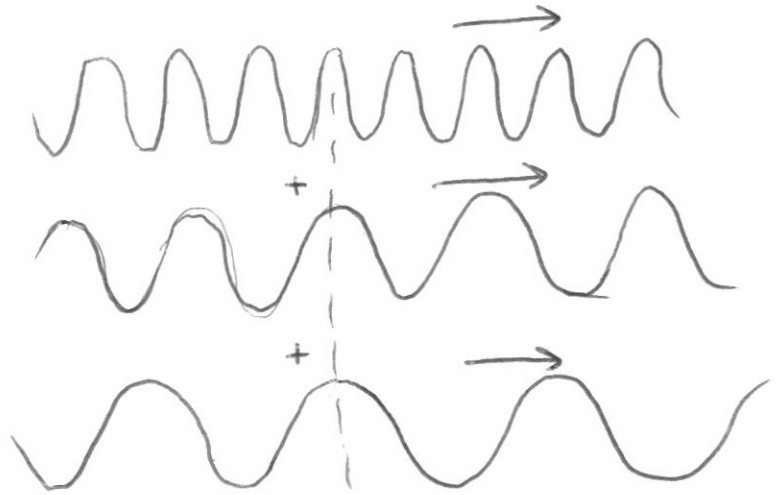
$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = ?$$

Fourier analysis
components



=



$$\sum_k c_k (B'_k \sin kx \cos \omega_k t - C'_k \cos kx \sin \omega_k t)$$

Formal way
of saying a superposition
of traveling waves that have
different k .

Conclusion: The formal mathematics works out

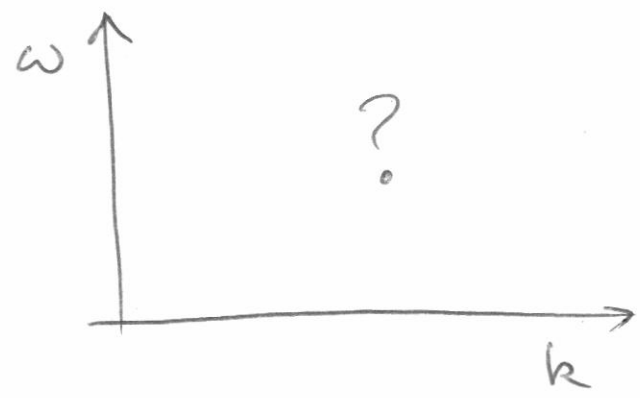
It describe all the possibilities we
observe in real life.

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Lab Exercise

Look at some real standing waves.
Each one will have different ω & k .

Natural question is



This is called a dispersion relation.

"Dispersion" is a very general word in physics
it just means "how one variable changes with another."

For today it will be sufficient to count # of nodes.

Data will be used in Friday hw.