DAY 2

PH424

webpage... show class notes from Day 1.

Pop Quiz

Last time: Phase velocity $v_{ph} = \frac{\omega}{k}$ $\left(= \frac{f}{\lambda} \right)$

Traveling waves always contain a $(kx - \omega t)$ term.

Write an expression for a parabola moving to the left.
$\psi(x,t) = (kx - \omega t)^2$

Demo: Standing wave. (Not all waves fns are traveling).

Write an expression describing a standing wave.

$$\boxed{\sin(kx + \phi_1) \sin(\omega t + \phi_2)}$$

Note that the function is separable

$$\psi(x,t) = X(x) T(t)$$

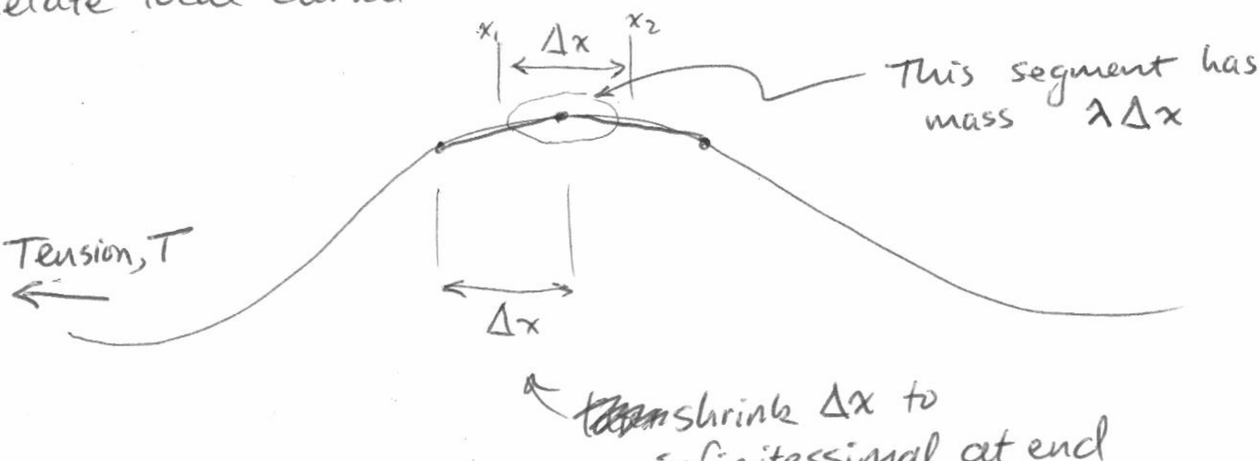
stretched strings are straight lines (in eq^b).

$$\frac{\partial^2 \psi}{\partial x^2} = 0 \qquad \text{where } \psi \text{ is displacement from eq}^b.$$

If $\frac{\partial^2 \psi}{\partial x^2} \neq 0$ there is a restoring force pushing string back to eq^b.

Therefore (by $F = ma$) there is accelleration.

Relate local curvature to local acceleration



Tension is pulling the segment down

$$F_z = T\left(\frac{\partial \psi}{\partial x}\bigg|_{x_1} - \frac{\partial \psi}{\partial x}\bigg|_{2} \right)$$

$$ma = T \frac{\partial^2 \psi}{\partial x^2} \Delta x$$

$$\boxed{\frac{\partial^2 \psi(x,t)}{\partial t^2} = \frac{T}{\lambda} \frac{\partial^2 \psi(x,t)}{\partial x^2}} \qquad \text{———} *$$

Wave eqn for string under tension T.
with linear mass density λ.

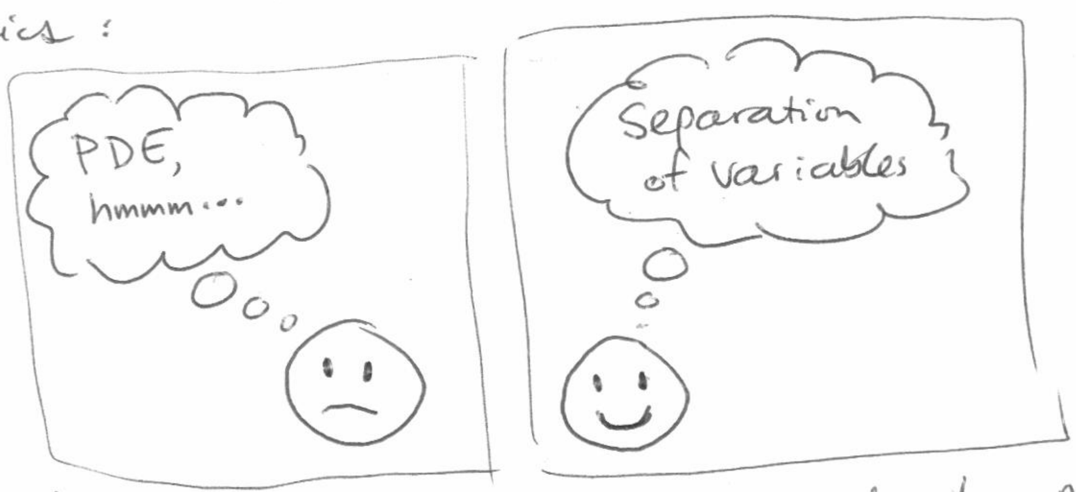
$\psi(x,t) = A \sin(kx - \omega t)$ satisfies $*$?

More examples in HW.

Yes, if $\frac{\omega}{k} = \sqrt{\frac{T}{\lambda}}$, the wave eqn is satisfied.

Guess and check is useful, but how do I find the most general solution to a 2nd order linear partial diff eqn?

Introduce a technique that comes up again & again in physics:



Because if S.o.V. doesn't work, your up the creek w/o a paddle

Get class to work thru this part

$$\frac{\partial^2}{\partial x^2}\Psi(x,t) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\Psi(x,t)$$

$$\Psi(x,t) \rightarrow X(x)T(t)$$

$$T(t)\frac{\partial^2}{\partial x^2}X(x) = \frac{1}{v^2}X(x)\frac{\partial}{\partial t^2}T(t)$$

$$\frac{1}{X(x)}\frac{\partial^2}{\partial x^2}X(x) = \frac{1}{v^2}\frac{1}{T(t)}\frac{\partial^2}{\partial t^2}T(t)$$

$$\boxed{\begin{aligned}\frac{1}{X(x)}\frac{d^2}{dx^2}X(x) &= -k^2 \\ \frac{1}{v^2}\frac{1}{T(t)}\frac{d^2}{dt^2}T(t) &= -k^2\end{aligned}}$$

[choosing a const that has the right dimensions]

Coupled ordininary 2nd order D.E.s ~~Much easier~~

Solutions are now tractable.

Note, k^2 could be positive or negative, and can take any value.

I'll focus on $k^2 > 0$.

Remember the general sol^n of a second order D.E. has two arbitrary constants and two linearly independent functions.

If $k^2 > 0$, then

$$X(x) = A \sin kx + B \cos kx$$

$$T(t) = C \sin \underbrace{kvt}_{=\omega} + D \cos \underbrace{kvt}_{=\omega}$$

$$\begin{aligned}\psi(x,t) = X(x)T(t) &= A' \sin kx \sin \omega t \\ &+ B' \sin kx \cos \omega t \\ &+ C' \cos kx \sin \omega t \\ &+ D' \cos kx \cos \omega t\end{aligned}$$

Slinky demo?

where k can have any value and $\omega = kv$ where v is const from the wave eqn.

? Are these traveling or standing waves?

If this is the general sol^n, how can we reconcile traveling wave sol^{n}s like

$$\psi(x,t) = A \sin(kx - \omega t) \quad ?$$

Computer demo

What about arbitrary pulse shapes?

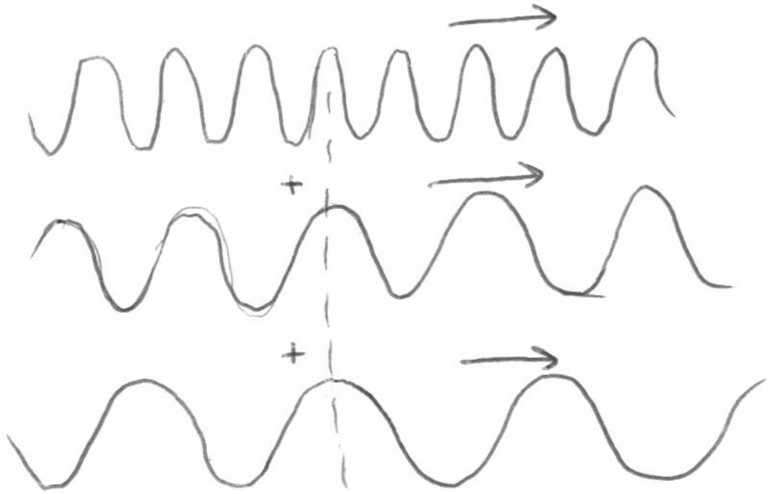
Computer demo

Trig identity $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \ ?$

Fourier ~~analysis~~ components

=



$$\underbrace{\sum_k c_k \left(B_k' \sin kx \cos \omega_k t - C_k' \cos kx \sin \omega_k t \right)}$$

Formal way of saying a superposition of traveling waves that have different k.

Conclusion: The formal mathmatics works out

It describe all the possibilities we obsene in real life.

Lab Exercise

Look at some real standing waves.

Each one will have different ω & k.

Natural question is

This is called a dispersion relation.

"Dispersion" is a very general word in physics

it just means "how one variable changes with another."

For today it will be sufficient to count # of nodes.

Data will be used in Friday hw.