

①

DAY 15

Quote concerning the Dirac eqn.  
- Nice motivation

### WKB Approximation



$$\frac{|F|^2}{|A|^2} = ?$$

First part of the approximation:  $C \gg D$

i.e. wave fn decays in the barrier.

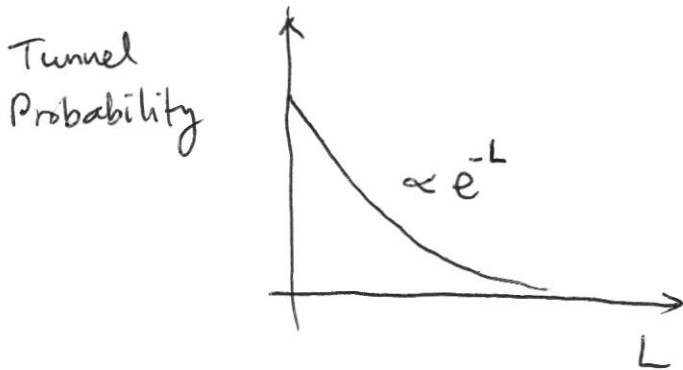
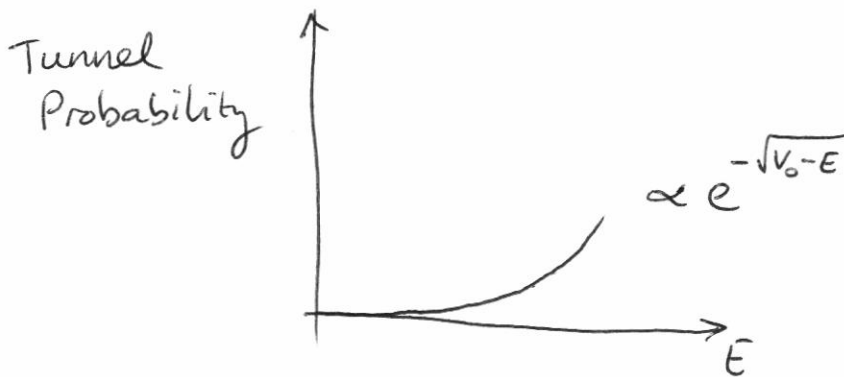
Second part of the approximation:

$$\frac{|F|^2}{|A|^2} \approx \frac{|\Psi_{\text{II}}(x=L)|^2}{|\Psi_{\text{II}}(x=0)|^2}$$

(2)

Applying these two approximations we find

$$\begin{aligned} \text{Transmission Prob} &= |e^{-k_2 L}|^2 \\ &= e^{-2k_2 L} \\ &= \exp\left(-\frac{2}{\hbar} \sqrt{2m(V_0 - E)} L\right) \end{aligned}$$



Compare to a recent research article, see link on website.

③

The Heisenberg uncertainty principle.

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Example 1: Use Mathematica to plot

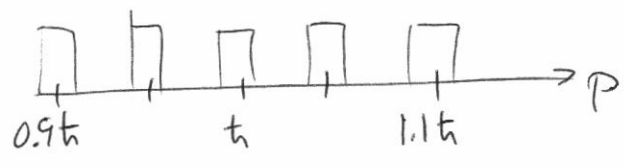
$$\text{Re} \left\{ e^{i(0.9)x} + e^{i(0.95)x} + e^{i(1.0)x} + e^{i(1.05)x} + e^{i(1.1)x} \right\}$$



see website.

What is  $\Delta p$ ?

Histogram of possible measurements



Standard deviation will be approximately  $0.05 h$ .

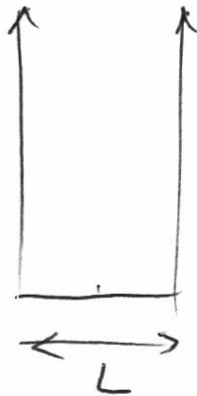
What is  $\Delta x$ ?

Standard deviation is approximately 10.

$$\Delta x \Delta p \approx \frac{\hbar}{2}$$

(4)

The uncertainty principle is a fantastic tool for making quick approximations in QM.



What is the smallest kinetic energy we could expect ~~an~~ bound electron to have in this well?

$$\Delta x \approx \frac{L}{4} \quad \text{i.e. } 60\% \text{ prob of finding } e^- \text{ between } \langle x \rangle \pm \frac{L}{4}.$$

$$KE \approx \frac{\Delta p^2}{2m}$$

$$\text{Uncertainty principle} \rightarrow \Delta p \approx \frac{\hbar}{2\Delta x} = \frac{\hbar}{2 \cdot \frac{L}{4}} = \frac{2\hbar}{L}$$

$$KE \approx \frac{\hbar^2 2^2}{2mL^2}$$

$$\text{Exact/Correct answer is } \frac{\hbar^2 \pi^2}{2mL^2}$$