DAY 14

PH424

Saturday Review 1.00pm - 2.30pm
Come with questions.

Hamiltonian for a free electron?

$$\hat{H} = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2}$$

Eigen functions of this Hamiltonian for a particular eigenvalue $E > 0$:

$$\phi(x,t) =$$

where $k = \sqrt{\frac{2mE}{\hbar^2}}$ & $\omega = \frac{E}{\hbar}$

Are there any restrictions on E?

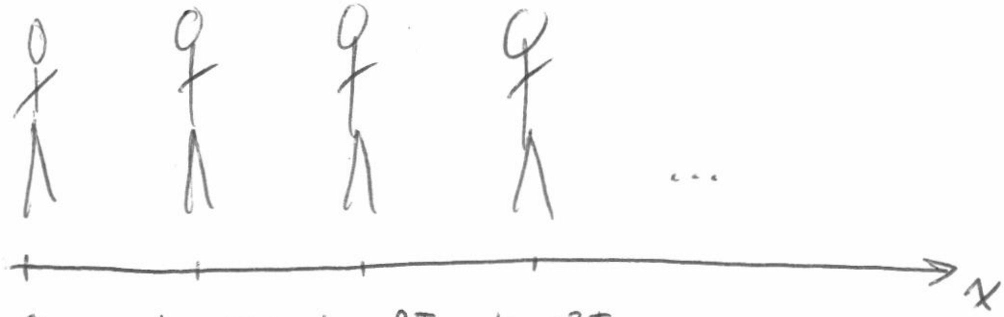
Can I normalize these ~~fun~~ eigenfns?

Either I will
- Do my calculations on a finite range of x.
- Only calculate ratios of wavefn amplitudes
- Work with wave packets instead of pure eigenfns of the Hamiltonian.

$\Psi(x,t) = e^{i(kx - \omega t)}$

$\boxed{t=0}$

e^{i0} $e^{i\frac{\pi}{4}}$ $e^{i\frac{2\pi}{4}}$ ...



0 $kx_1 = \frac{\pi}{4}$ $kx_2 = \frac{2\pi}{4}$ $kx_3 = \frac{3\pi}{4}$ ...

$\boxed{\omega t = \frac{\pi}{4}}$

$e^{-i\frac{\pi}{4}}$ e^{i0} $e^{i\frac{\pi}{4}}$ ...

continue the animation.

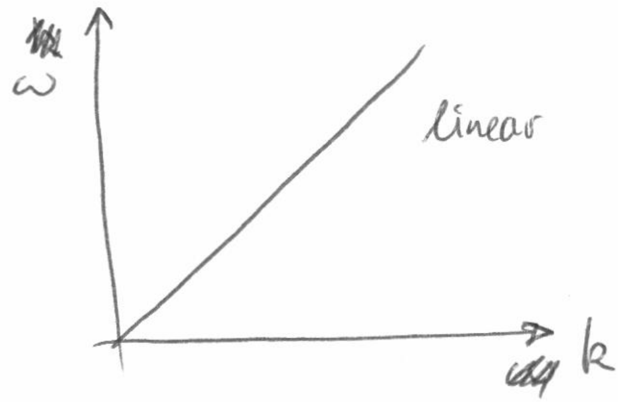
What happens if I double the wave vector of this state?

Ans ω gets four times bigger.

Remember $k = \sqrt{\dfrac{2mE}{\hbar^2}}$ $\omega = \dfrac{E}{\hbar}$

Therefore $\omega = \dfrac{\hbar k^2}{2m}$

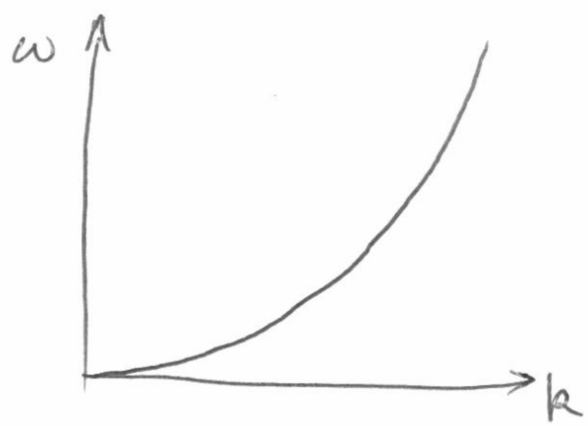
Recall waves on a stretched string



Non-disperse.

i.e. V_{phase} does not change with k.

Compare to a quantum particle



Dispersive,

V_{phase} changes with k.

~~Alternatively~~.

The S. Eq^n is a dispersive wave eq^n.

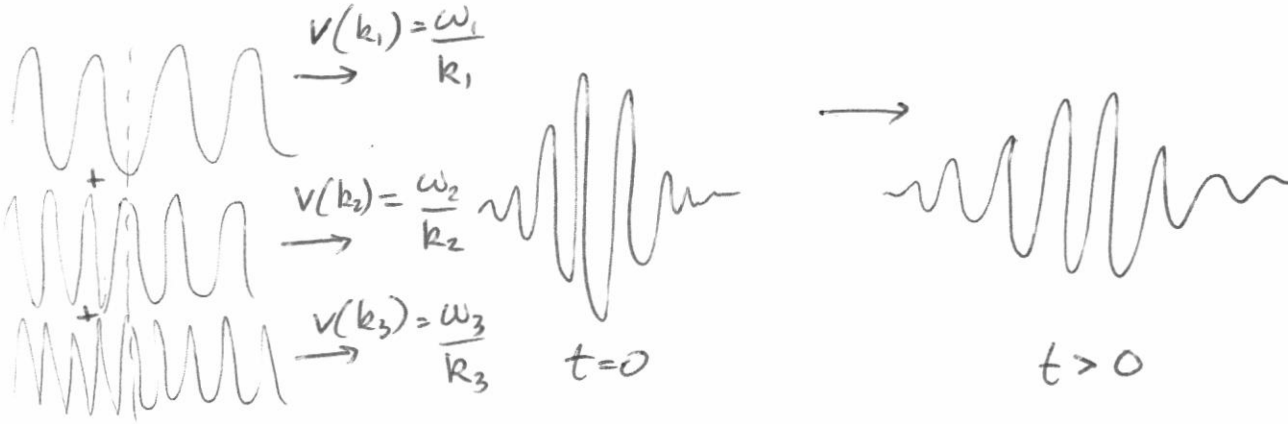
~~The phas~~ V_{phase} changes with wave vector.

$\boxed{\text{we know that electron velocity should be } \frac{p}{m} = \frac{\hbar k}{m} \left(= V_{phase} ? \right)}$

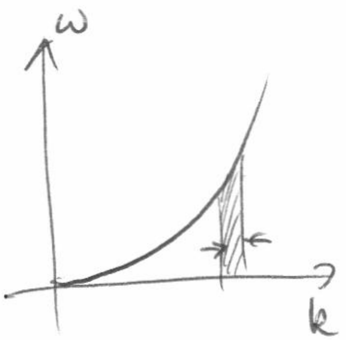
Consequences ?

① Wave packets won't ~~stay~~ hold their shape.

small range of wave vectors Δk.



② The envelope of the wave packet moves at a different speed than the phase velocity.



Consider the center line where constructive interference is happening. Motion of this center line depends on the ~~[illegible]~~ differences in V_{ph} between diff component.

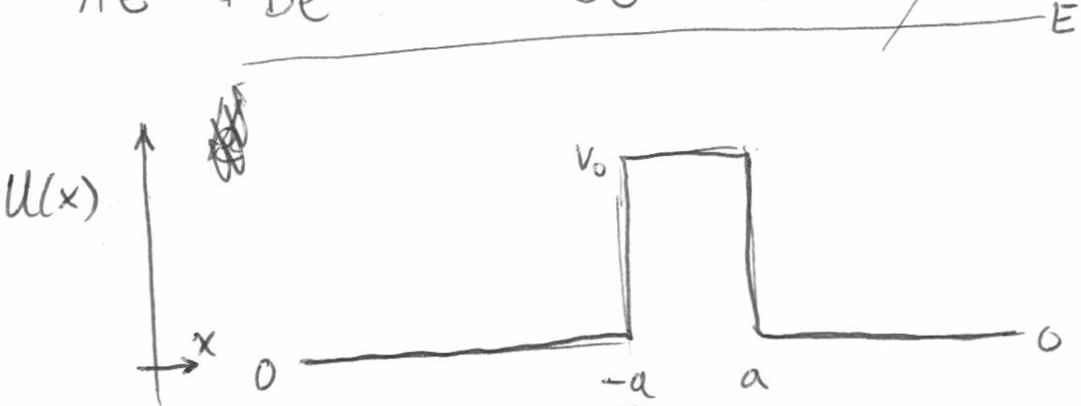
$$V_{group} = \frac{d\omega}{dk}.$$

check the value of V_{group} for an electron wave.

I II 4 III

Study this situation in steady state

$Ae^{ik_1x} + Be^{-ik_1x}$ $\qquad Ce^{ik_2x} + De^{-ik_2x}$ $\qquad \cancel{Ee^{-ik_1x}}^{\,0} + Fe^{ik_1x}$



~~Const~~ Calculate the energy eigenstate at $t=0$ ~~for~~ when source of the particles is on the left and system has reached steady state.

$\boxed{k_1 = \sqrt{\frac{2mE}{\hbar^2}}}$ $\qquad \boxed{k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}}$

A, B, C, D, F ~~might~~ could be complex #s if the interfaces introduce phase shifts.

at $x = -a$, boundary conditions

$$\Rightarrow \quad Ae^{-ik_1a} + Be^{ik_1a} = Ce^{-ik_2a} + De^{ik_2a}$$

$$\& \quad ik_1Ae^{-ik_1a} - ik_1Be^{ik_1a} = ik_2Ce^{-ik_2a} - ik_2De^{ik_2a}$$

at $x = +a$

$\Rightarrow$