$$|\Phi\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle$$

Energy Eigenstates of the infinite square well potential

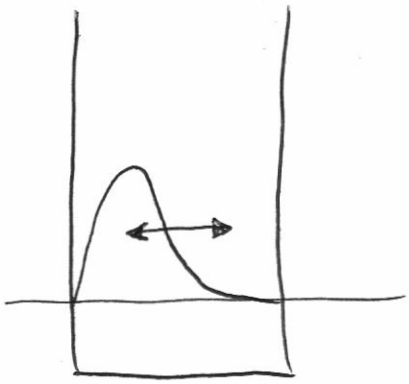
To explicity write down $\Phi(x,t)$ I need to know E_1, E_2, $\phi_1(x)$, $\phi_2(x)$ in terms of the width of the well and the mass of the particle.

Review this calculation

Now
$$|\Phi(x,t)|^2 = \frac{1}{2}|\phi_1(x)|^2 + \frac{1}{2}|\phi_2(x)|^2 + \frac{1}{2}\phi_1^*(x)\phi_2(x)e^{\frac{i}{\hbar}(E_2-E_1)t} + \frac{1}{2}\phi_2^*(x)\phi_1(x)e^{\frac{i}{\hbar}(E_1-E_2)t}$$

$$= \underbrace{\frac{1}{2}\phi_1^2(x) + \frac{1}{2}\phi_2^2(x)}_{\text{Doesn't change with time}} + \underbrace{\phi_1(x)\phi_2(x)\cos\left(\frac{E_2-E_1}{\hbar}\right)t}_{\text{\underline{Does} change with time.}}$$

Animation shows



Probability density moves back and forth at $\omega = \frac{\Delta E}{\hbar}$.

Note that an electron oscillating at ω can radiate light with freq ω and photon energy $\hbar\omega$.

This is our first glimpse of how excited atoms can emit radiation at quantized energies.

Energy Eigenstates

$|\phi_n(x,t)|^2$ is stationary

Super position states

$|\Psi(x,t)|^2$ is <u>not</u> stationary.

Question : For $\frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle$,

$$\left.\begin{array}{l}\langle \hat{x} \rangle \\ \langle \hat{p} \rangle \\ \langle \hat{E} \rangle\end{array}\right\}$$ which are time dependent ?

Answer

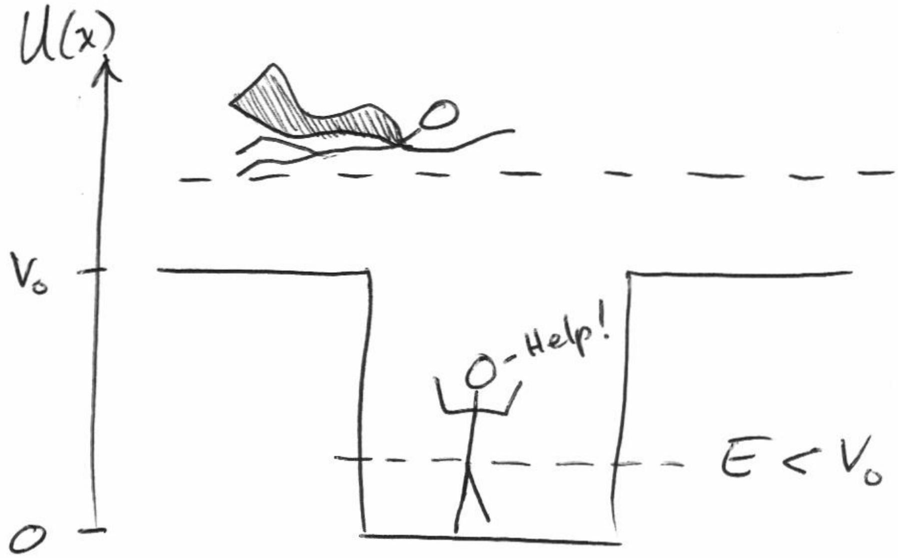
$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^* x \, \psi \, dx = \underbrace{\int_{-\infty}^{\infty} x \, |\psi|^2 dx}$$

weighted average.
Definitely changes, because $|\psi|^2$ is changing.

$\langle p \rangle$; If $\langle x \rangle$ is changing then $\langle p \rangle$ must be changing.

$\langle E \rangle$; Energy conservation says it should not change with time.

Calculate $\langle E \rangle = \frac{1}{2} E_1 + \frac{1}{2} E_2 = \text{const.}$



unbound states (Continuum of energies)

Bound states. (Discrete levels)

Analogy

Stretched string with no end. Any wavevector k is possible.

Guitar string with clamped ends.

$$k = \frac{n\pi}{L} \qquad n = 1, 2, 3 \ldots$$