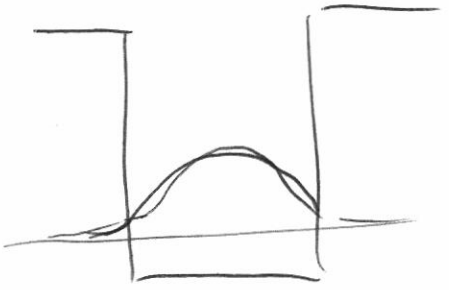
DAY 11

Ph424

Last time: Detailed solution to the finite square well.



First exercise: Quick estimate of the lowest eigen energy.

Task: Check that the PhET is giving a physically reasonable result for lowest energy state.

Soln: In the well $U(x) = 0$ and $\psi(x) = A\cos kx$

Eigenvalue eqn is $\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$

$k = \frac{\pi}{a}$

$$\Rightarrow E = \frac{\hbar^2 k^2}{2m}$$

$$= \frac{\hbar^2 \pi^2}{2ma^2}$$

i.e. Zero potential energy, plus this much kinetic energy.

$$= \frac{(10^{-34})^2 10}{2\ 10^{-30}(10^{-9})^2} = \frac{10^{-68}\ 5}{10^{-30}\ 10^{-18}}$$

$$= \frac{5 \cdot 10^{-50}}{10^{-48}} = 5 \cdot 10^{-20}\ \text{J}$$

$$\frac{0.5 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.3 \text{ eV}$$

WHERE WE ARE NOW:

- We know what a wavefn represents

  i.e. Wave function is "continuum limit of a wave vector.

- We know (in principle) how to find energy eigenfunctions and eigenvalues.

  i.e. Solve the eigenvalue eqn and find possible values for the unspecified constant "E."

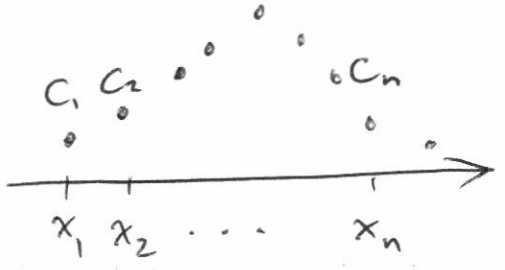
OTHER COMMON QUANTIES WE MIGHT WANT TO KNOW:

- Expectation values
- Projections (for example, probability that E_1 is measured)
- Spread in measured values.

These calculations are a natural extension of your previous work with vectors.

> Functions are the continuum limit of vectors

vector (list of #s)



function

Dot product between two vectors

$$\begin{bmatrix} v_1^* & v_2^* & \cdots & v_N^* \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}$$

$$= \sum_{i=1}^{N} v_i^* c_i$$

"Dot product" between two functions

$$\int_{-\infty}^{\infty} v^*(x)\, c(x)\, dx$$

Both operations are called

$$\langle v | c \rangle$$

Practice translating "bra-ket" notation into wave function integrals.

$$\langle \psi | H | \psi \rangle = ?$$

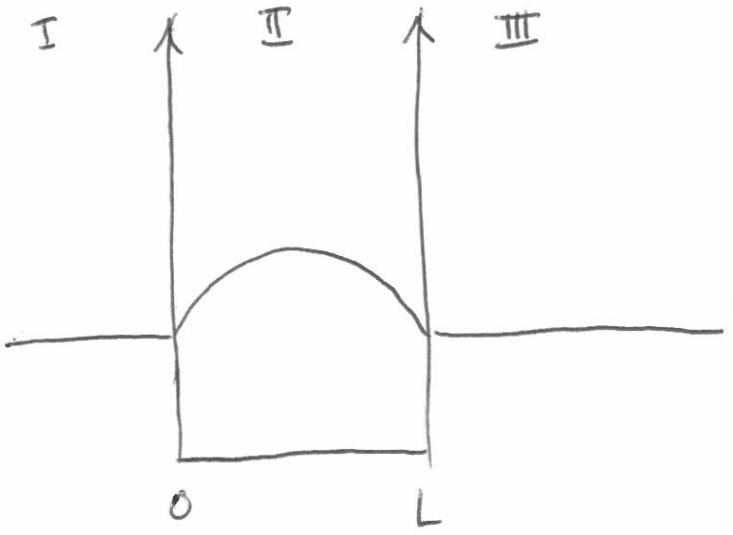
$$= \int_{-\infty}^{\infty} \psi^*(x) \hat{H} \psi(x) \, dx$$

↑ differential operator

$$\langle \psi | \psi \rangle = ?$$

$$= \int_{-\infty}^{\infty} \psi^*(x) \psi(x) \, dx$$

$$= \int_{-\infty}^{\infty} |\psi(x)|^2 \, dx$$



$$\Psi(x) = \begin{cases} 0 & \text{I} \\ A \sin \frac{\pi}{L} x & \text{II} \\ 0 & \text{III} \end{cases}$$

Find the normalization constant A.

Answer $A = \sqrt{\frac{2}{L}}$