DAY 10

PH 424

Questions about HW?

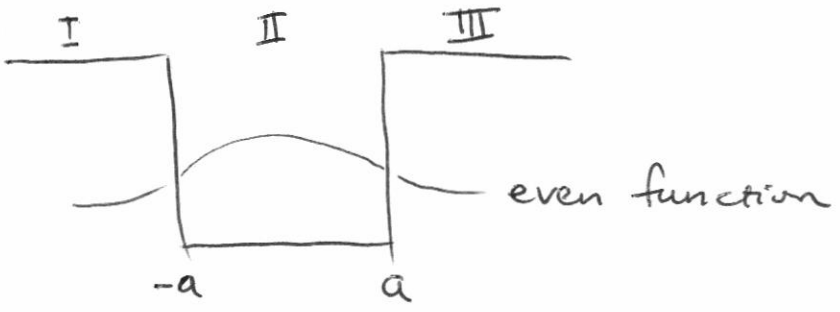
Last time: Energy Eigenfunctions & eigenvalues for a finite square well.

Review by looking at PhET simulator (see class website link).

- Finite number of bound states
- Eigen energies change as function of width and well depth.
- Even, odd, even, odd pattern to the functions.
- Lower energy states have less wavefunction curvature $\left(\frac{\partial^2\psi}{\partial x^2}\right)$ in the classically allowed region.
- The probability distribution, $|\Psi(x)|^2$, has the same symmetry as $U(x)$.

SMALL GROUP ACTIVITY:

Given the following information, find possible values for E, the eigen energies.



$$\phi_E = \begin{cases} A e^{k_1 x} & \text{I} \\ B \cos k_2 x & \text{II} \\ C e^{-k_1 x} & \text{III} \end{cases}$$

where $k_1 = \sqrt{\dfrac{2m(V_0 - E)}{\hbar^2}}$

$k_2 = \sqrt{\dfrac{2mE}{\hbar^2}}$

Solution: Focus on the interface between region I & II.

$\dfrac{d\phi_E}{dx}$ & ϕ_E must be continuous

$$\Rightarrow \quad B \cos k_2 a = C e^{-k_1 a} \qquad \text{——①}$$

$$-k_2 B \sin k_2 a = -k_1 C e^{-k_1 a} \qquad \text{——②}$$

Dividing these equations leads to

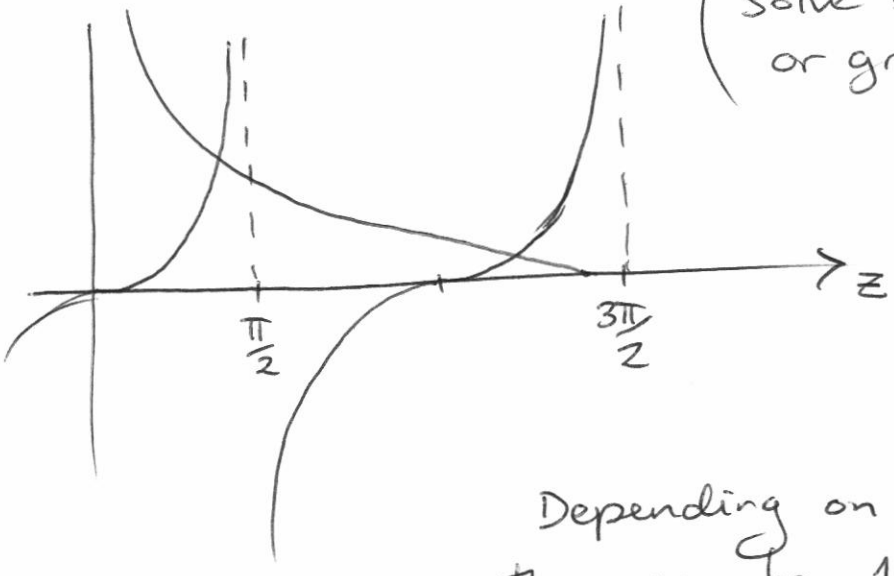
$$\tan k_2 a = \frac{k_1 a}{k_2 a}$$

$$\tan\sqrt{\frac{2mEa^2}{\hbar}} = \sqrt{\frac{\frac{2mV_0a^2}{\hbar^2} - \frac{2mEa^2}{\hbar^2}}{\frac{2mEa^2}{\hbar^2}}}$$

let $z = \sqrt{\frac{2mEa^2}{\hbar^2}}$, if we can find z, we've solved the problem.

$$\tan z = \sqrt{\frac{\text{const} - z^2}{z^2}}$$

(Transcendental eqn. Solve by computer, or graphically.)



Depending on the const, there can be 1, 2, 3... even eigenfunctions that satisfy the system.

Note: An infinite potential well corresponds to const $\to \infty$. Then $z = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ etc.

The well known solutions of an infinite square well potential.