Homework #2 (Lab)

Parts 1-4 due Wednesday 02/13/13,
Parts 5-6 due Wednesday 02/20/13.
Turn in either a hardcopy or by email

This homework assignment is designed to take you through the experiment you conducted with pulses in a coaxial cable, and to work through each step again, thinking through the physical model and the experimental data. You are expected to write good prose, valid equations, and to pay attention to clarity. Be succinct, but make sure you explain clearly and demonstrate steps in important derivations. Your audience is incoming juniors majoring in physics who do not know the experiment or the model, not instructors. If you work through this worksheet, you will have written a good lab report (except that in a lab report, you would put question 6 first). If there are parts of the experiment you rushed through, or you were not the oscilloscope "driver", spend a few minutes setting up the experiment again. It will take only 20 minutes the second time through!

1. **Methods:** Briefly describe the experiment you did, including an informative circuit diagram. Include separate sketches of the oscilloscope trace for important values the terminating resistor. [10 points. Points are assigned for correct results and for your ability to put your discussion in a sensible, clear and logical fashion.]

2. **Presentation of results:** Present your results for the measurement of the wave speed and for the measurement of the pulse height as a function of terminating resistance in tabular and/or graphical form as appropriate, and include a succinct verbal description (in complete sentences, please) that explains what you are presenting. (This does not mean interpretation of the results. It means explaining in words that augment the tables and graphs.) [20 points. Points are assigned for clarity of presentation, and for a succinct and accurate description, appropriateness of number of points and information given, and whether the results are valid.]

3. **Interpret wave speed results:** Based on your experimental result for the propagation speed, and the results of homework 1, question 7, predict a value for the dielectric constant of the insulator in the coaxial cable you used. Do a little research to find typical values for such insulators and compare. [10 points]

4. **Model for propagation:** Based on the analogies in Main, Table 10.1, we see that the non-dispersive wave equation for "charge" $\psi(x,t)$ in a cable is $\frac{1}{C_0}\frac{\partial^2\psi}{\partial x^2} = L_0\frac{\partial^2\psi}{\partial t^2}$, where C_0 is the capacitance per unit length and L_0 is the inductance per unit length. If $\psi(x,t)$ represents charge, then $\frac{\partial\psi}{\partial t}$ represents current, and $\frac{\partial\psi}{\partial x}$ represents the charge density, which is proportional to voltage, V (charge/length = $C_0 * V$).

(a) What is the propagation velocity?

(b) Suppose two cables in which waves propagate with different speeds are connected. Given that $\psi_1(x,t) = e^{i(k_1x-\omega t)} + Re^{i(-k_1x-\omega t)}$ represents a possible solution in cable 1 that is a sum of an incident and reflected charge wave, and $\psi_2(x,t) = Te^{i(k_2x-\omega t)}$ is a transmitted wave in a cable, re-derive the values of R and T based on appropriate boundary conditions at the join. However, you did NOT measure charge in your experiment, you measured voltage, which is proportional to $\frac{\partial \psi}{\partial x}$.

(c) Derive reflection & transmission coefficients for the <u>voltage</u> wave that propagates in the cable.

(d) Define a quantity called the *impedance* Z in each cable: $Z \equiv \frac{V}{I}$. Using the representations above, show that $Z = \frac{k/C_0}{\omega}$ and hence rewrite the reflection and transmission coefficients in terms of Z_1 and Z_2. Also show that $Z = \sqrt{\frac{L_0}{C_0}}$.

(e) Based on your results in (c) predict the height of the voltage pulse at the termination end of the cable, and the height of reflected pulse at the source end (relative to the incident pulse)? (e) Reproduce the data plot in (2) and graph the result of the calculation on the same plot. Comment on the agreement or lack thereof between the model and the data. [20 points]

5. **Refined model:** If the coaxial cable wires have internal resistance per unit length R_0, the electrons lose energy as they oscillate about their local equilibrium point. In this case, the wave equation has an additional "frictional force" term: $\frac{1}{L_0 C_0}\frac{\partial^2 \psi}{\partial x^2} - \frac{R_0}{L_0}\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial t^2}$ that comes from damping in the cable. (This should remind you of the LRC circuit equation we studied in PH421).

(a) Show that, <u>if the resistive term is small</u>, then the solution to the damped equation has a similar form to the undamped situation $\psi(x,t) = Ae^{i(k_c x - \omega t)}$ *except that* k_c is a complex number $k + i\kappa$, and $k = \frac{\omega}{v} = \omega\sqrt{L_0 C_0}$ and $\kappa = \frac{R_0}{2vL_0}$.

(b) What is the significance of the imaginary part of k ? *i.e.* How does it affect the shape of the waveform and is its physical significance?

(c) Now think about your experiment. What is the consequence of this new term in the equation of motion? Reproduce the data plot in (2) and graph the result of the new calculation on the same plot. Comment on the agreement or lack thereof between the model and the data. [25 points]

6. **Summary**: Write a short summary (<200 words) that summarizes the results of the experiment, the modeling that you did, and the validity of the model. [5 points]