PH 315 Homework for Week 3
Model Answers

Problem #1:

a) I will use the idea of "area under the curve". There are two approaches I can take.

First, I can count boxes. Each box on the picture is a rectangle of "height" = 50 kPa, "width" = 0.02 m^3, and "area" = 1 kJ. Approximately 14 boxes fit under the curve. Therefore,

$$W = -\int_{0.02\ \mathrm{m}^3}^{0.1\ \mathrm{m}^3} P dV \approx -14\ \mathrm{kJ}.$$

Alternatively, the curve looks like the top half of an ellipse. The area of an ellipse is πab where *a* is the minor axes radius and *b* is the major axis radius. In this case *a* = 100 kPa and *b* = 0.04 m^3, so the area of half this ellipse is 6.3 kJ. Add to this the rectangular area underneath the half ellipse to find

$$W = -\int_{0.02\ \mathrm{m}^3}^{0.1\ \mathrm{m}^3} P dV = -(6.3\ \mathrm{kJ} + 8\ \mathrm{kJ}) = -14.3\ \mathrm{kJ}.$$

This second method is more precise, but the first method gave a correct answer within ±5% absolute error.

b) The functional form for pressure will be easy to integrate using calculus. The initial conditions, P_i and V_i, can be used to find a value for the constant:

$$\mathrm{constant} = \alpha = P_\mathrm{i} V_\mathrm{i}^{5/3} = 2.15 \times 10^3\ \mathrm{Pa \cdot m^5}$$

Thus, the work is

$$W = -\alpha \int_{0.1\ \mathrm{m}^3}^{0.05\ \mathrm{m}^3} V^{-\frac{5}{3}}\, dV = -[2.15 \times 10^3\ \mathrm{Pa \cdot m^5}]\left(-\frac{3}{2}\right)\left(V_\mathrm{f}^{-2/3} - V_\mathrm{i}^{-2/3}\right)$$

Plugging in values, the work done on the gas is $W = +8.81$ kJ. By convention, we denote this as positive work because the work energy flowed into the gas as it was compressed.

c) Numerical integration: The quickest and easiest method is to add the 8 values of pressure and then multiply by the step size in volume:

$$W = -\left(\sum_{i=1}^{i=8} P_i\right) \cdot \Delta V$$

where $\Delta V = 0.05$ liters and P_i is the i^{th} value of pressure. This approach gives

W = -755 kPa.liters = -755 J.

By convention, we denote this as negative work. The work energy flowed out of the gas as it expanded. (The sign tells you which way to draw the arrow in an energy flow diagram).

Problem #2. Miscellaneous

a) I'll assume ideal gas behavior (which is an excellent assumption, especially at when there are very few molecules occupying a large volume). Therefore, I expect $P = Nk_BT/V$.
We are told $N = 1$ when V = 1 cm^3 = 10^{-6} m^3, and the temperature is 3 K (i.e. the molecules have very little kinetic energy). The pressure is approximately 4 x 10^{-17} Pa. This is more than 1000 times lower than what humans have achieved in the laboratory.

b) Imagine a container with one movable wall, e.g. the cylinder and piston of an engine. Imagine the container is full of gas molecules, which bounce around inside like little rubber balls unaffected by gravity. Whenever a ball hits a stationary wall, it bounces back with the same speed (elastic collisions). Now, imagine that the piston is moving slowly away from the gas-filled chamber, i.e. the gas is expanding. When the ball bounces off that moving wall, it recoils with a slower speed. This reduces the kinetic energy of the ball, thus the temperature of the gas is lower. If the piston is moving into the chamber, then each molecule that hits the piston recoils at a faster speed. The gas gets hotter.

Problem #3 Specific heat capacity

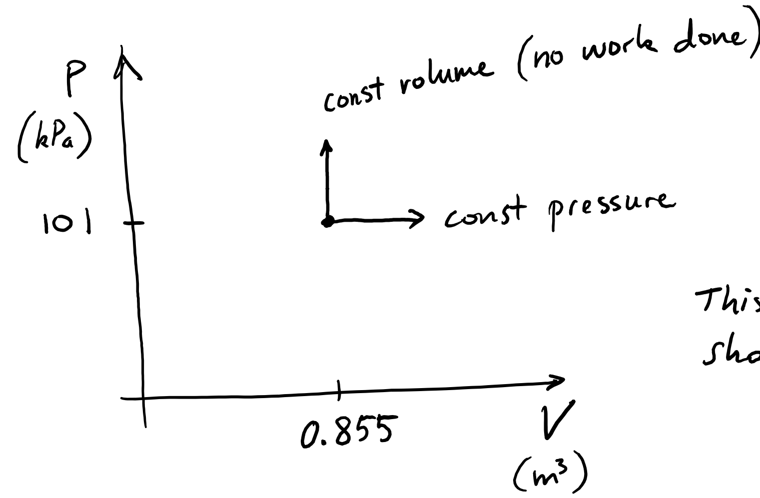
(see next page, handwritten)

HW3 cv cp

Monday, October 21, 2019 8:57 PM

I want to check $c_p = 1.00 \frac{kJ}{kg \cdot K}$ and $c_v = 0.72 \frac{kJ}{kg \cdot K}$ for air at standard conditions. I'll assume the air is 100% N_2.

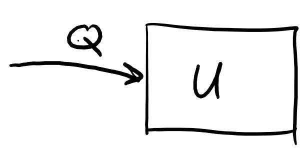
a)



The number of molecules is $N = \frac{1000\,g}{28\,g/mol} \times 6.02 \times 10^{-23}$

$$\approx 214 \times 10^{23}$$

b) For the constant volume process



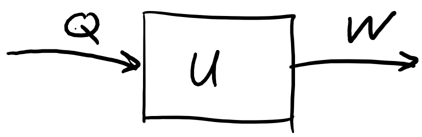
Heat goes into the gas. No energy leaves the gas.

$$U = \frac{5}{2} N k_B T$$

$$Q = \frac{5}{2} N k_B \Delta T = \frac{5}{2} (214 \times 10^{23})(1.38 \times 10^{-23} \tfrac{J}{K})(1\,K)$$

$$= 0.735\ kJ$$

c) For the constant pressure process



Heat goes into the gas. Work leaves the gas.

$$|W| = \int P\,dV = (P)(\Delta V) \quad \text{because } P \text{ is constant.}$$

To find ΔV, note that

$$V = \frac{N k_B T}{P} \quad \text{since we keep } N \;\&\; P \text{ constant.}$$

Therefore $\Delta V = \frac{N k_B}{P} \Delta T$

This implies that

$$|W| = N k_B \Delta T$$
$$= (2.14 \times 10^{23})(1.38 \times 10^{-23} \tfrac{J}{K})(1\,K)$$
$$= 0.294\ kJ$$

To hit my temperature target of 1 K increase, I need U to increase by 0.735 kJ, therefore

$$Q = 0.735\,kJ + 0.294\,kJ$$
$$= 1.03\ kJ$$

c) The heat values that I calculated are very close to the values predicted by c_v and c_p (0.72 kJ and 1.00 kJ). My numbers are a few percent on the high side. When I assumed 100% N_2, it caused a slight overestimate of the number of diatomic molecules in air (the molecular weight of oxygen is 32 g/mol).

Problem #4: Degrees of Freedom
Postposed until next homework.

Problem #5: Earthquake Warning System
I want to know how much time I have to prepare before an earthquake arrives in Corvallis.

I'll Consider the extreme cases:
Maximum speeds: p-wave speed is 5 km/s, s-wave speed is 3 km/s.
Minimum speeds: p-wave speed is 3 km/s, s-wave speed is 1.8 km/s.

(a) Newport is about 50 miles away, and from the picture it looks like the subduction zone is another 50 miles away. I'll assume the total distance from Corvallis to the subduction zone is 150 km. Using the relationship between velocity, distance and time ($v = d/t$), I calculate the minimum and maximum travel times for the P-waves and S-waves:

	Newport	Corvallis
P-waves	15-25 seconds	30-50 seconds
S-waves	25-42 seconds	50-83 seconds

In Corvallis, the time delay between p-waves and s-waves is between 20-33 seconds. If you're fast you might make it out of the building if you start moving when you feel the p-wave. However, not everyone would make it out.

(b) First assume the max speeds. The p-wave reaches Newport at 15 s, Corvallis gets the s-wave at 50 s. This would give you 35 s to escape the building. (I've assumed instantaneous communication between Newport and Corvallis because the speed of light waves is 10^5 times faster than the speed of earthquake waves).

Now assume the minimum speeds. Newport gets the p-wave 25 s, Corvallis gets the s-wave at 83 s. This would give me 58 s to escape the building.