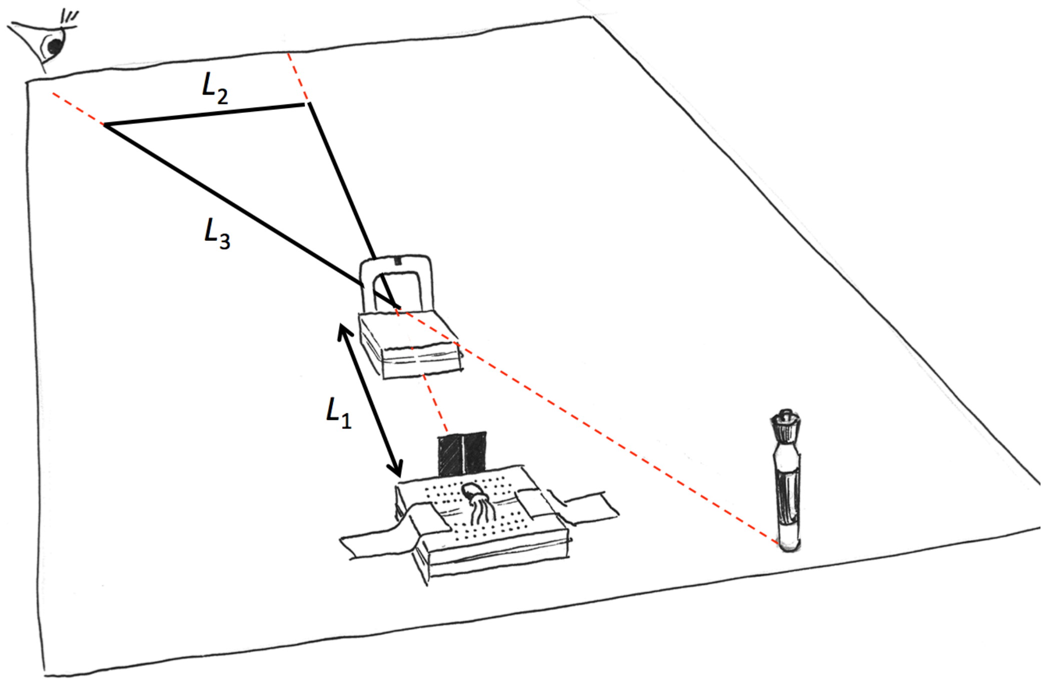
Homework #6
Due on February 24 at 5pm. Hand in to Paul Emigh (office 491 Weniger).

1. Experimental skills



You are collecting data to find the wavelength of light, λ, from an LED. The diffraction grating (1000 lines/mm) is a distance L_1 from the collimating slit. You find the position of the diffracted image and draw lines on the white board. You measure the lengths L_2 and L_3. You are confident in you length measurements to within ±0.2 cm.

You repeat the experiment 3 times. For each replica of the experiment, you erase the lines on the white board, move the diffraction grating to a slightly different position, and find the new position of the image. Here are the final results:

	L_1 (cm)	L_2 (cm)	L_3 (cm)
Expt 1	26	28.5	42.5
Expt 2	28	27.5	40.5
Expt 3	30	27	41

a) For experiment 1, what is the percentage uncertainty in L_1, L_2 and L_3? For example 10 ± 1 cm would correspond to an uncertainty of 10%.

b) What range of λ is consistent with the lengths reported in the first row of the table (Expt 1)? Remember that the actual lengths could be 0.2 cm longer or shorter than reported in the table.

c) Using your answer to part b, express the uncertainty in λ as a percentage. How does this relate to the percentage uncertainties you found in part a?

d) What wavelengths are consistent with the numbers recorded in Expt 2 and 3?

e) Using the estimated wavelengths from experiment 1, 2 and 3 (i.e. three different numbers) calculate the standard deviation.

It is possible that the accuracy of the ruler is not the only source of experimental uncertainty. For example, the 90-degree corner of the L_2-L_3 triangle might actually be 90 ±5 degree. Therefore, we want to compare the standard deviation seen across multiple experimental trials (part e) to the uncertainty that is expected in a single trial due to uncertainty in L_1, L_2 and L_3.

f) Compare the standard deviation seen across multiple experimental trials (the standard deviation calculated in part e) to the uncertainty that is expected in a single trial due to uncertainty in L_1, L_2 and L_3. Do you think that L_1, L_2 and L_3 are the dominant source of "random noise" in the experiment, or should the experimenters look for other sources of "random noise" that are meddling with their experiment?

g) A different group of students designs their experiment so that all distances are twice as long. They measure L_1 = 42 ±0.2 cm, L_2 = 57 ±0.2 cm, L_3 = 85 ±0.2 cm. What is the range of λ that is consistent with this group's measurement? Give your answer as a percentage and compare to part c.

2. Interference

Sound waves with a frequency of 320 Hz are emitted by two speakers 44 cm wide and 3.5 m apart. Find the angles of the lines along which the waves from the speakers constructively interfere (assuming that wave crests are emitted by the speakers simultaneously).

3. Photons

a) Imagine that you are facing an LED light bulb that emits 10 W of electromagnetic radiation (with wavelengths between 400 – 700 nm). The light bulb is 3 m away from you. If the diameter of your pupils is about 2 mm, about how many photons enter your eye every second?

b) Experiments have shown that the nervous system of the human eye effectively takes about 30 "frames" per second (like a movie camera) and that when the eye is fully dark-adapted, it needs to receive only about 500 photons per frame from an object to register it. Our sun radiates a power of about 3.9 x 10^{26} W at all wavelengths (only about half of this energy is in the visible range, however). Estimate how far away a star like the sun could be and still be visible.

4. Multiplexing light on a fiber optic cable

Each high-performance optical fiber that carries information around the internet carries light of approximately 71 different wavelengths:
1530.0 nm, 1530.5 nm, 1531.0 nm, 1531.5 nm… up to 1565 nm.

Each wavelength, or "color", carries an independent signal (switching on and off at about 100 GHz = 10^{11} times per second). Different "colors" of light share the same optical fiber, without interfering with one another.

a) Each bit of information is transmitted in about 10^{-11} seconds. About how many times does the electric field of the light cycle up and down during 10^{-11} seconds?

b) Global internet traffic is approximately 640 x 10^{18} bits of information per month. If each bit is carried by a pulse of 10,000 photons, calculate a lower bound for the energy per person per day required to run the internet. (Assume there are 500 million internet users). Give your answer in kWh.

When the light arrives at the end of the fiber optic cable, the different colors of light must be spread out from one another. You are given a diffraction grating with 300 lines per mm, and told to spread out the 71 different "colors" of light on a detector array. Each color will illuminate a separate spot on the detector. The center-to-center spacing between spots should be 40 μm.

c) Draw a diagram showing how you will use the diffraction grating the spread out the different colors of light on the detector array. Make sure to specify the distance between the diffraction grating and the detector array.

The commercial solution to this challenge involves arrayed waveguide gratings.
https://en.wikipedia.org/wiki/Arrayed_waveguide_grating

5. Stretching a CO_2 molecule

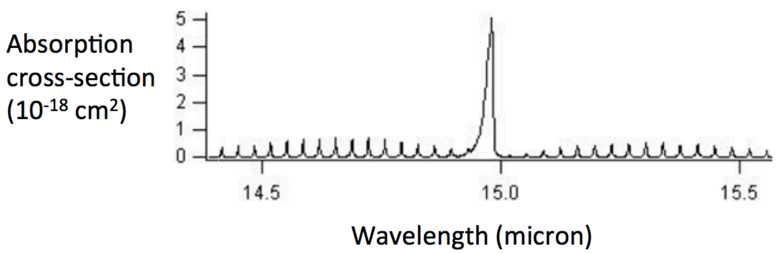
The equilibrium length of a C=0 bond is 0.123 nm. The spring constant of a C=O bond is 10^3 N/m. Assume that the net charge associated with the carbon atom is +e and the net charge associated with each oxygen atom is -0.5e.
What is the change in bond length when a constant, uniform electric field $E_0 = 10^6$ V/m is applied along the long axis of the CO_2 molecule?

Note: The Coulomb force between effective charges is already included in the spring constant. Therefore, this problem only involves an electric force on each atom ($F = qE_0$) and the tension in each spring.

6. Scattering length

The graph below shows experimental measurements of the absorption cross-section of CO_2 as a function of wavelength

http://vpl.astro.washington.edu/spectra/co2pnnlimagesmicrons.htm



a) The CO_2 concentration in our atmosphere is currently 406 ppm (winter 2017), i.e. 0.04% of the gas molecules are CO_2. In the lower atmosphere (near sea level), how many CO_2 molecules are there per cubic meter?

b) Consider a photon traveling through the lower part of our atmosphere. Assume that the wavelength of the photon is on-resonance with this peak in the CO_2 absorption cross-section. How far can the photon travel before it has about a 50% chance of being scattered by a CO_2 molecule? I'm looking for an answer in units of meters.