From Monday:

To get wave-like and discrete behavior for particles like electrons we need both a wave equation and boundary conditions. Let's look at the simple standing wave case to illustrate why this behavior makes sense.

For a wave on a string, the displacement $y(x,t)$ is governed by:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\mu}{T} \frac{\partial^2 y(x,t)}{\partial t^2}$$

where $\frac{T}{\mu}$ is a constant equal to the wave speed squared (v^2)

What do we mean by boundary conditions? What are they here?

$$y(0,t) = 0 \quad \text{and} \quad y(L,t) = 0$$

I claim that ~~y(x,t) = 5 cm~~.

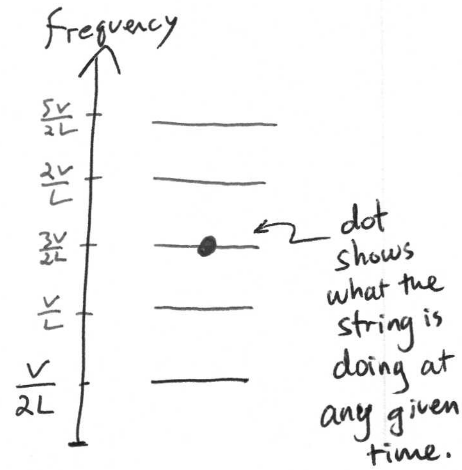
$$y(x,t) = y_0 \sin\left[\frac{\pi x}{L}\right] \sin\left[\frac{\pi V}{L} t\right]$$

is a solution that meets both conditions. Let's check!

What is the frequency of this wave?

$$\omega = \frac{\pi V}{L} \qquad f = \frac{\omega}{2\pi} = \frac{V}{2L}$$

We found one solution! It might not be the only one, though, so we'll keep track of all possible solutions on the figure below, which we will call a frequency level diagram.



What are some other possible solutions to this system?

Because of the general shape of a diagram like this, we might also call it a frequency level diagram.

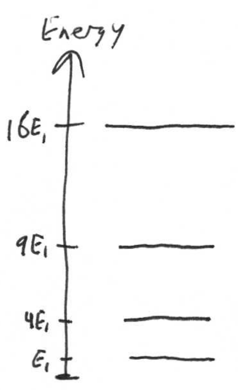
The string we saw yesterday was only vibrating at one frequency at a time. We can represent the particular frequency by placing a dot on the appropriate level. What would it mean if we put two dots on different levels? What does it mean that the diagram stays the same regardless of where we put a dot?

We are interested in particles, not waves on a string. We will therefore draw <u>Energy Level Diagrams</u> to show the different possible (discrete) energies available to an electron in some system.

Draw an energy level diagram for the analogous electron in a 1D-box from yesterday, which has energy values,

$E_n = n^2 E_1$, where E_1 is constant

(we'll assume E_1 is 1 eV)



Recall from Day 22, ~~this~~ the set of energies $E_n = n^2 E_1$

where $n = 1, 2, 3, 4 \ldots$ comes from solving

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

with the boundary conditions $\Psi(x=0) = 0$ and $\Psi(x=L) = 0$.

The mathematics (a partial differential equation with boundary conditions) is analogous to solving

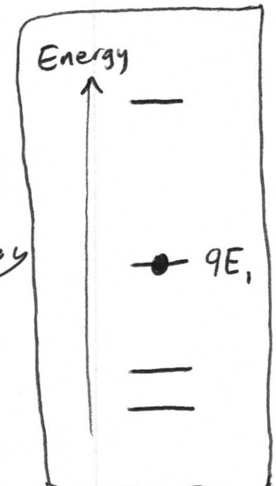
$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2}$$

with boundary conditions

$y(x=0) = 0$

$y(x=L) = 0.$

In both cases we get a discrete set of solutions.



Dot shows what the electron is doing at any given time

Now suppose we know that an electron in this system has energy $E_3 = 9E_1$. A photon hits this electron — how much energy could this photon have if the electron absorbs it?

Suppose it ends up with $E_4 = 16E_1$ after absorbing the photon.

1. Draw an energy level diagram showing the change.
2. Find the wavelength of the photon.

The electron hangs out at E_4 for awhile, then spontaneously falls all the way to $E_2 = 4E_1$.

1. Draw an energy level diagram showing the change.
2. What photon must be emitted?

In your groups, find all possible wavelengths of visible light that can be emitted or absorbed by a hydrogen atom, which has energy levels $E_n = -\frac{13.6 \text{ eV}}{n^2}$

Draw energy level diagrams supporting your answers.

Important note: The energy level diagram shows the energy levels of the <u>electron</u>, not the photons. What part of the diagram ends up corresponding to photon energies?