PH 652 Quantum Mechanics 29 January 2016

Homework 3

Due Wednesday 10 February

1. Use your favorite software tool to plot the two-particle probability density for two non-interacting particles in a 1-d harmonic oscillator potential for the case where one of the particles is in the single-particle ground state $|0\rangle \doteq \varphi_0(x)$ and the other in the first excited state $|1\rangle \doteq \varphi_1(x)$. Do this for the three cases of (a) distinguishable particles (of the same mass) (b) identical particles in a symmetric spatial state and (c) identical particles in an antisymmetric spatial state. In each case, write the system wave function and discuss the important features of your plots.

2. Calculate the one-dimensional particle separation probability density $\mathcal{P}(x_1 - x_2)$ for a system of two identical particles in a 1-d harmonic oscillator potential with one particle in the single-particle ground state $|0\rangle \doteq \varphi_0(x)$ and the other in the first excited state $|1\rangle \doteq \varphi_1(x)$. Show graphically how you calculate $\mathcal{P}(x_1 - x_2)$ from $\mathcal{P}(x_1, x_2)$. Calculate and plot $\mathcal{P}(x_1 - x_2)$ for the three cases of (a) distinguishable particles (of the same mass) (b) identical particles in a symmetric spatial state and (c) identical particles in an antisymmetric spatial state. Discuss.

3. a) Show that if an observable commutes with the Hamiltonian, then that observable is conserved (i.e. its expectation value does not change with time)
b) Show that if an operator representing a physical observable A is unchanged by a unitary transformation U, then A and U commute.
c) An infinitesimal unitary transformation of magnitude ε can be written as

$$U(\varepsilon) = I - \frac{i\varepsilon}{\hbar} G$$

where G is the generator of the transformation. Show that if an operator representing a physical observable A is unchanged by an infinitesimal unitary transformation U, then A and G commute.