## PH481 Homework 4

Due: Monday, $6^{\text {th }}$ of February 2023
7.4* Show that the optical path length, defined as the sum of the products of the various indices times the thicknesses of media traversed by a beam, that is, $\sum_{i} n_{i} x_{i}$, is equivalent to the length of the path in vacuum that would take the same time for that beam to negotiate.
7.6* Determine the optical path difference for the two waves $A$ and $B$, both having vacuum wavelengths of 500 nm , depicted in Fig. P.7.6; the glass ( $n=1.52$ ) tank is filled with water $(n=1.33)$. If the waves start out in-phase and all the above numbers are exact, find their relative phase difference at the finishing line.

7.7* Using Eqs. (7.9), (7.10), and (7.11), show that the resultant of the two waves
and

$$
E_{1}=E_{01} \sin [\omega t-k(x+\Delta x)]
$$

$$
E_{2}=E_{01} \sin (\omega t-k x)
$$

is $\quad E=2 E_{01} \cos \left(\frac{k \Delta x}{2}\right) \sin \left[\omega t-k\left(x+\frac{\Delta x}{2}\right)\right]$
You could also do this problem using the complex exponential forms of the waves. That is, let

$$
\begin{aligned}
& E_{1}=E_{01} e^{i[\omega t-k(x+\Delta x)]} \\
& E_{2}=E_{02} e^{i[\omega t-k x]}
\end{aligned}
$$

and then use the imaginary part to get the sine functions as needed.
7.54 Write an expression for the transform $A(\omega)$ of the harmonic pulse of Fig. P.7.54. Check that sinc $u$ is $50 \%$ or greater for values of $u$ roughly less than $\pi / 2$. With that in mind, show that $\Delta v \Delta t \approx 1$, where $\Delta v$ is the bandwidth of the transform at half its maximum amplitude. Verify that $\Delta v \Delta t \approx 1$ at half the maximum value of the power spectrum as well. The purpose here is to get some sense of the kind of approximations used in the discussion.

Figure P.7.54

7.63* Suppose we spread white light out into a fan of wavelengths by means of a diffraction grating and then pass a small select region of that spectrum out through a slit. Because of the width of the slit, a band of wavelengths 1.2 nm wide centered on 500 nm emerges. Determine the frequency bandwidth and the coherence length of this light.

