PH 451: Capstone in Quantum Mechanics

Homework 8

Due 3/1/10

1. Calculate the size of the following energy terms and spin orbit and relativistic corrections for the hydrogen atom (for a-d, tabulate your results and give answers in three forms: theoretical in terms of $\alpha^n mc^2$, in *eV* or *meV*, and in *GHz*)

a) The energy difference between the n = 1 and n = 2 states BEFORE any perturbations were considered.

b) The correction to the n = 1 and n = 2 states due to spin-orbit coupling. Note that the formula we derived in class is problematic for $\ell = 0$. Show that if you set $j = \ell + \frac{1}{2}$ and then use $j = \frac{1}{2}$, the problem goes away. (This is the Darwin term we talked about, but go ahead and call it spin-orbit here.)

c) The correction to the n = 1 and n = 2 states due to the relativistic term.

d) The total correction to these states, *i.e.,* the fine structure correction.

e) What wavelength resolution must your detector have to be able to able to resolve the two lines in the n = 2 to n =1 transition? Be careful here. When you include the correction, you will find that it is very small compared to the unperturbed value. Be sensible about how to include the effects.

f) Is it important to use the reduced mass of the electron in your calculations or is it OK to use the free mass?

2. In the intermediate magnetic field regime where the Zeeman corrections and the fine structure corrections are comparable, we have to treat both effects as perturbations to the zeroth-order Bohr energy levels.

a) The Hamiltonian $H' = H'_{fs} + H'_Z$ in the uncoupled basis $|n\ell s m_\ell m_s\rangle$, is given in Eqn. 14.82 in the text. Diagonalize this Hamiltonian and produce the plot in Fig. 14.10.

b) Find the Hamiltonian $H' = H'_{fs} + H'_Z$ in the in the coupled basis $|n\ell s j m_j\rangle$. Diagonalize and show that the results agree with part (a).

3. a) Find the hyperfine structure of the ground state of deuterium. The gyromagnetic ratio of the deuteron is $g_D = 0.857$, which is the only change needed in Eqn. 13.49. Find the splitting of the ground state and produce a figure like Fig. 13.4.

b) Solve for the Zeeman splitting of the ground state of deuterium in intermediate fields and produce a figure like Fig. 14.11.