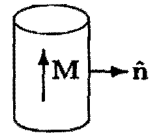
Problem 6.1

$$\mathbf{N} = \mathbf{m}_2 \times \mathbf{B}_1;\ \mathbf{B}_1 = \frac{\mu_0}{4\pi}\frac{1}{r^3}[3(\mathbf{m}_1 \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}_1];\ \hat{\mathbf{r}} = \hat{\mathbf{y}}; \mathbf{m}_1 = m_1\hat{\mathbf{z}};\ \mathbf{m}_2 = m_2\hat{\mathbf{y}}.\quad \mathbf{B}_1 = -\frac{\mu_0}{4\pi}\frac{m_1}{r^3}\hat{\mathbf{z}}.$$

$\mathbf{N} = -\frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^3}(\hat{\mathbf{y}}\times\hat{\mathbf{z}}) = -\frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^3}\hat{\mathbf{x}}$. Here $m_1 = \pi a^2 I$, $m_2 = b^2 I$. So $\boxed{\mathbf{N} = -\frac{\mu_0}{4}\frac{(abI)^2}{r^3}\hat{\mathbf{x}}.}$ Final orientation : $\boxed{\text{downward}}$ $(-\hat{\mathbf{z}})$.

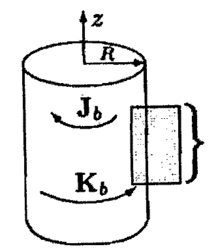
Problem 6.7



$\mathbf{J}_b = \nabla\times\mathbf{M} = 0$; $\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} = M\hat{\boldsymbol{\phi}}$.

The field is that of a surface current $\mathbf{K}_b = M\hat{\boldsymbol{\phi}}$, but that's just a solenoid, so the field $\boxed{\text{outside is zero,}}$ and inside $B = \mu_0 K_b = \mu_0 M$. Moreover, it points upward (in the drawing), so $\boxed{\mathbf{B} = \mu_0\mathbf{M}.}$

Problem 6.12



(a) $\mathbf{M} = ks\hat{\mathbf{z}}$; $\mathbf{J}_b = \nabla\times\mathbf{M} = -k\hat{\boldsymbol{\phi}}$; $\mathbf{K}_b = \mathbf{M}\times\hat{\mathbf{n}} = kR\hat{\boldsymbol{\phi}}$.
B is in the z direction (this is essentially a superposition of solenoids). So $\boxed{\mathbf{B} = 0 \text{ outside.}}$ Use the amperian loop shown (shaded)—inner side at radius s:
$\oint \mathbf{B}\cdot d\mathbf{l} = Bl = \mu_0 I_{\text{enc}} = \mu_0\left[\int J_b\, da + K_b l\right] = \mu_0\left[-kl(R-s) + kRl\right] = \mu_0 kls.$
$\therefore \boxed{\mathbf{B} = \mu_0 ks\hat{\mathbf{z}} \text{ inside.}}$

(b) By symmetry, **H** points in the z direction. That same amperian loop gives $\oint \mathbf{H}\cdot d\mathbf{l} = Hl = \mu_0 I_{f_{\text{enc}}} = 0$, since there *is* no free current here. So $\boxed{\mathbf{H} = 0}$, and hence $\boxed{\mathbf{B} = \mu_0\mathbf{M}.}$ *Outside* $\mathbf{M} = 0$, so $\mathbf{B} = 0$; *inside* $\mathbf{M} = ks\hat{\mathbf{z}}$, so $\mathbf{B} = \mu_0 ks\hat{\mathbf{z}}$.

Problem 6.25

(a) $\mathbf{B}_1 = \frac{\mu_0}{4\pi}\frac{2m}{z^3}\hat{\mathbf{z}}$ (Eq. 5.86, with $\theta = 0$). So $\mathbf{m}_2\cdot\mathbf{B}_1 = -\frac{\mu_0}{2\pi}\frac{m^2}{z^3}$. $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$ (Eq. 6.3) $\Rightarrow$ $\mathbf{F} = \frac{\partial}{\partial z}\left[-\frac{\mu_0}{2\pi}\frac{m^2}{z^3}\right]\hat{\mathbf{z}} = \frac{3\mu_0 m^2}{2\pi z^4}\hat{\mathbf{z}}$. This is the magnetic force *upward* (on the upper magnet); it balances the gravitational force downward $(-m_d g\hat{\mathbf{z}})$:

$$\frac{3\mu_0 m^2}{2\pi z^4} - m_d g = 0 \Rightarrow \boxed{z = \left[\frac{3\mu_0 m^2}{2\pi m_d g}\right]^{1/4}.}$$

(b) The middle magnet is repelled *upward* by lower magnet and *downward* by upper magnet:

$$\frac{3\mu_0 m^2}{2\pi x^4} - \frac{3\mu_0 m^2}{2\pi y^4} - m_d g = 0.$$

The top magnet is repelled *upward* by middle magnet, and attracted *downward* by lower magnet:

$$\frac{3\mu_0 m^2}{2\pi y^4} - \frac{3\mu_0 m^2}{2\pi (x+y)^4} - m_d g = 0.$$

Subtracting: $\frac{3\mu_0 m^2}{2\pi}\left[\frac{1}{x^4} - \frac{1}{y^4} - \frac{1}{y^4} + \frac{1}{(x+y)^4}\right] - m_d g + m_d g = 0$, or $\frac{1}{x^4} - \frac{2}{y^4} + \frac{1}{(x+y)^4} = 0$, so: $2 = \frac{1}{(x/y)^4} + \frac{1}{(x/y+1)^4}$.
Let $\alpha \equiv x/y$; then $2 = \frac{1}{\alpha^4} + \frac{1}{(\alpha+1)^4}$. Mathematica gives the numerical solution $\alpha = \boxed{x/y = 0.850115\ldots}$