Problem 3.27

$\mathbf{p} = (3qa - qa)\,\hat{\mathbf{z}} + (-2qa - 2q(-a))\,\hat{\mathbf{y}} = 2qa\,\hat{\mathbf{z}}$. Therefore

$$V \cong \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2},$$

and $\mathbf{p}\cdot\hat{\mathbf{r}} = 2qa\,\hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 2qa\cos\theta$, so

$$V \cong \boxed{\frac{1}{4\pi\epsilon_0}\frac{2qa\cos\theta}{r^2}.} \quad \text{(Dipole.)}$$

Problem 3.28

(a) By symmetry, $\mathbf{p}$ is clearly in the z direction: $\mathbf{p} = p\,\hat{\mathbf{z}}$; $p = \int z\rho\,d\tau \Rightarrow \int z\sigma\,da$.

$$\begin{aligned} p &= \int (R\cos\theta)(k\cos\theta)R^3\sin\theta\,d\theta\,d\phi = 2\pi R^3 k\int_0^\pi \cos^2\theta\sin\theta\,d\theta = 2\pi R^3 k\left(-\frac{\cos^3\theta}{3}\right)\Big|_0^\pi \\ &= \frac{2}{3}\pi R^3 k[1-(-1)] = \frac{4\pi R^3 k}{3}; \quad \boxed{\mathbf{p} = \frac{4\pi R^3 k}{3}\,\hat{\mathbf{z}}.} \end{aligned}$$

(b)

$$V \cong \frac{1}{4\pi\epsilon_0}\frac{4\pi R^3 k}{3}\frac{\cos\theta}{r^2} = \boxed{\frac{kR^3}{3\epsilon_0}\frac{\cos\theta}{r^2}.} \quad \text{(Dipole.)}$$

This is *also* the *exact* potential. *Conclusion:* all multiple moments of this distribution (except the dipole) are exactly zero.

Problem 4.10

(a) $\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}} = \boxed{kR;}$ $\rho_b = -\nabla\cdot\mathbf{P} = -\frac{1}{r^3}\frac{\partial}{\partial r}(r^2 kr) = -\frac{1}{r^2}3kr^2 = \boxed{-3k.}$

(b) For $r < R$, $\mathbf{E} = \frac{1}{3\epsilon_0}\rho r\,\hat{\mathbf{r}}$ (Prob. 2.12), so $\mathbf{E} = \boxed{-(k/\epsilon_0)\,\mathbf{r}.}$

For $r > R$, same as if all charge at center; but $Q_{tot} = (kR)(4\pi R^2) + (-3k)(\frac{4}{3}\pi R^3) = 0$, so $\boxed{\mathbf{E} = 0.}$

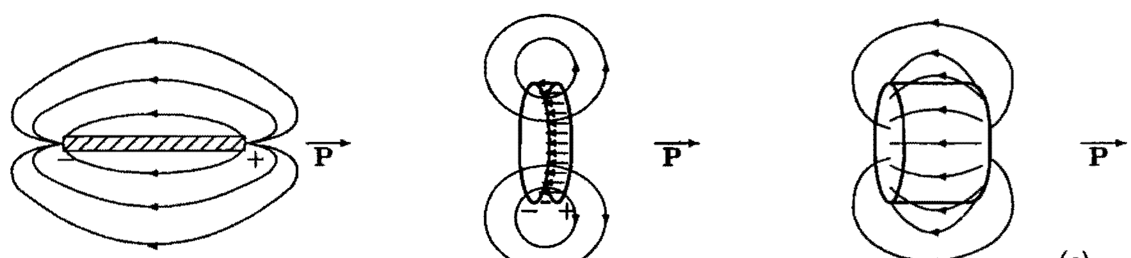
Problem 4.11

$\rho_b = 0$; $\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}} = \pm P$ (plus sign at one end—the one $\mathbf{P}$ points *toward*; minus sign at the other—the one $\mathbf{P}$ points *away* from).

(i) $L \gg a$. Then the ends look like point charges, and the whole thing is like a physical dipole, of length L and charge $P\pi a^2$. See Fig. (a).

(ii) $L \ll a$. Then it's like a circular parallel-plate capacitor. Field is nearly uniform inside; nonuniform "fringing field" at the edges. See Fig. (b).

(iii) $L \approx a$. See Fig. (c).



(a) Like a dipole (b) Like a parallel-plate capacitor (c)