Ph 431 Electromagnetism Capstone Fall 2009

MIDTERM EXAM – November 2, 2009

This exam is closed book and closed notes except for the equation sheet provided. Please do all your work in the blue books. **Only the blue books will be graded!**

There are 3 main parts with a total of 100 points possible on this exam.

Budget your time wisely! Not all questions are of equal difficulty.

1. A point charge $+q$ is placed a distance a above an infinite grounded horizontal conducting plane. A second point charge $-q$ is placed directly above the first charge at a distance $(a + d)$ above the conductor. **(35 points)**
   a) Find the leading contribution to the electric potential at a distant (i.e., $r \gg a, d$) point above the surface of the conductor (i.e., on the same side as the charges).
   b) Find the exact electric field at an arbitrary point at the surface (i.e., just above) of the conductor (i.e., on the same side as the charges).
   c) Find the induced surface charge density (Coulombs/square meter) at an arbitrary point on the surface of the conductor.

2. A solid cylinder has a uniform polarization frozen into it. The polarization is characterized by an electric dipole moment density $\vec{\mathbf{P}}$, which is parallel to the cylinder axis and constant throughout the cylinder. This polarization is not caused by an applied field, but does produce its own electric field. **(35 points)**

   a) Show that the lines of $\vec{\mathbf{D}}$ are bent as they pass through the curved cylindrical surface and that they are not bent at the flat end faces.
   b) Show that the lines of $\vec{\mathbf{E}}$ are not bent as they pass through the curved cylindrical surface and that they are bent at the flat end faces.

3. A solid sphere of radius R has a uniform magnetization frozen into it. The magnetization is characterized by a magnetic dipole moment density (magnetization) $\vec{\mathbf{M}}$, which is constant throughout the sphere. This magnetization is not caused by an applied field, but does produce its own magnetic field. Inside the sphere, the magnetic field due to the magnetization is uniform and given by $\vec{\mathbf{B}} = 2\mu_0 \vec{\mathbf{M}}/3$. **(30 points)**

a) Find the magnetic field at the outside surface of the sphere (i.e., just outside).

b) Show that the magnetic field at the outside surface of the sphere has the same form as the magnetic field that would be produced there by a pure dipole $\vec{\mathbf{m}}$ at the origin. By equating the two fields, determine the dipole moment of the polarized sphere.