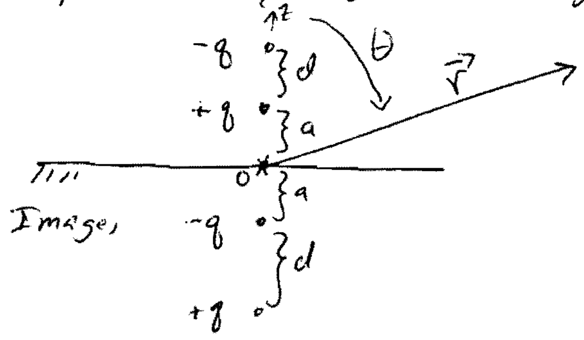
1. Potential & field above conductor are same as produced by original charges & image charges



i	q_i	θ_i'	r_i'
1	$-q$	θ	$a+d$
2	$+q$	θ	a
3	$-q$	$\pi-\theta$	a
4	$+q$	$\pi-\theta$	$a+d$

a) Use multipole expansion

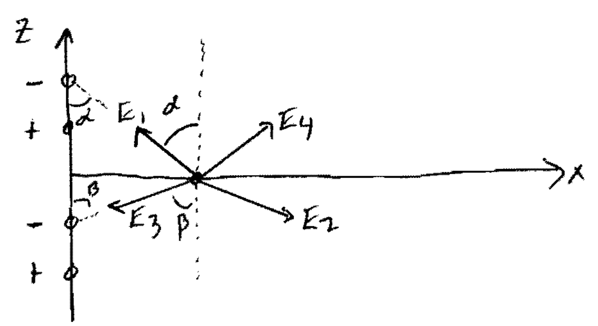
$$V = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \sum_i (r_i')^n P_n(\cos\theta_i)\, q_i$$

$$n=0: \quad \frac{1}{4\pi\epsilon_0}\frac{1}{r}(-q+q-q+q) = 0$$ (no monopole term)

$$n=1: \quad \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\left[-q(a+d)\cos\theta + q(a)\cos\theta - q(a)\cos(\pi-\theta) + q(a+d)\cos(\pi-\theta)\right]$$

$$\Rightarrow V(r,\theta) = \frac{1}{4\pi\epsilon_0}\frac{(-2qd\cos\theta)}{r^2}$$ dipole is leading term

b) Exact $\vec{E}$ by summing terms



$$\vec{E} = \sum_i \vec{E_i}$$

$$= (2E_1\cos\alpha - 2E_2\cos\beta)\,\hat{z}$$

$$E_1 = \frac{q}{4\pi\epsilon_0}\frac{1}{\rho_1^2}, \quad E_2 = \frac{q}{4\pi\epsilon_0}\frac{1}{\rho_2^2}$$

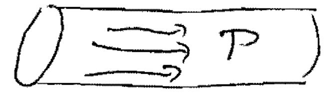
$$\rho_1^2 = x^2+(a+d)^2 \qquad \rho_2 = x^2+a^2$$

$$\cos\alpha = \frac{a+d}{\rho_1} \qquad \cos\beta = \frac{a}{\rho_2}$$

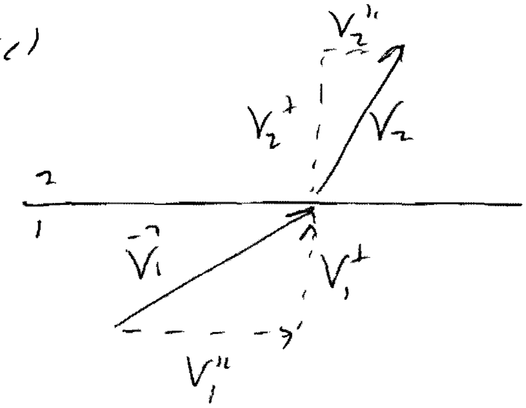
$$\vec{E} = \frac{2q}{4\pi\epsilon_0}\left[\frac{a+d}{(x^2+(a+d)^2)^{3/2}} - \frac{a}{(x^2+a^2)^{3/2}}\right]\hat{z} \qquad E = \frac{\sigma}{\epsilon_0}$$

$$\sigma = \frac{2q}{4\pi}\left[\frac{a+d}{(x^2+(a+d)^2)^{3/2}} - \frac{a}{(x^2+a^2)^{3/2}}\right]$$

2. Use boundary conditions at surfaces to find relative angles of $\vec{D}, \vec{E}$



In general



If $\dfrac{V^{\parallel}}{V_{\perp}}$ same on 2 sides, then $\vec{V}$ is not bent & vice versa

B.C. $\quad E_{in}^{\parallel} = E_{out}^{\parallel}$

$D_{in}^{\perp} = D_{out}^{\perp}$ since $\sigma_f = 0$

$E_{out}^{\perp} = E_{in}^{\perp} + \dfrac{\sigma}{\epsilon_0}$ $\quad \sigma = \sigma_b = \vec{P} \cdot \hat{n}$

$D_{out}^{\parallel} - D_{in}^{\parallel} = P_{out}^{\parallel} - P_{in}^{\parallel} = -P^{\parallel}$

End Caps: $P_{\parallel} = 0$, $\vec{P} \cdot \hat{n} = \pm P$ on 2 ends

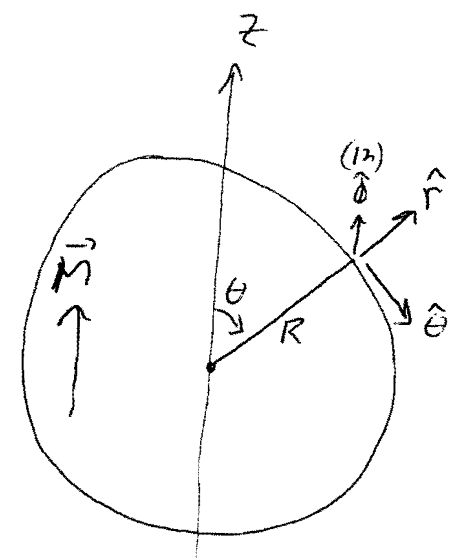
$$\Rightarrow \quad \frac{D_{out}^{\parallel}}{D_{out}^{\perp}} = \frac{D_{in}^{\parallel}}{D_{in}^{\perp}} \Rightarrow \vec{D} \text{ not bent}$$

$$\frac{E_{out}^{\parallel}}{E_{out}^{\perp}} = \frac{E_{in}^{\parallel}}{E_{in}^{\perp} \pm \frac{P}{\epsilon_0}} \neq \frac{E_{in}^{\parallel}}{E_{in}^{\perp}} \Rightarrow \vec{E} \text{ bent}$$

Sides: $P_{\parallel} = P$, $\vec{P} \cdot \hat{n} = 0$

$$\Rightarrow \quad \frac{D_{out}^{\parallel}}{D_{out}^{\perp}} = \frac{D_{in}^{\parallel} - P}{D_{in}^{\perp}} \neq \frac{D_{in}^{\parallel}}{D_{in}^{\perp}} \Rightarrow \vec{D} \text{ bent}$$

$$\frac{E_{out}^{\parallel}}{E_{out}^{\perp}} = \frac{E_{in}^{\parallel}}{E_{in}^{\perp}} \Rightarrow \vec{E} \text{ not bent}$$



$\underline{3.}$ $\vec{B}_{in} = \frac{2}{3}\mu_0 \vec{M}$

a) $\vec{H}_{in} = \frac{1}{\mu_0}\vec{B} - \vec{M} = -\frac{1}{3}\vec{M}$

Use boundary conditions at surface

$B^{\perp}_{in} = B^{\perp}_{out}$, $\vec{H}^{\parallel}_{in} = \vec{H}^{\parallel}_{out}$ $(\vec{K}_f = 0)$

$\vec{B}^{\parallel}_{out} - \vec{B}^{\parallel}_{in} = \mu_0(\vec{K}_b \times \hat{n})$, $H^{\perp}_{out} - H^{\perp}_{in} = m^{\perp}$

$B^{\perp}_{in} = B_{in,r} = \vec{B}_{in}\cdot\hat{r} = \frac{2}{3}\mu_0 M\cos\theta \Rightarrow B_{out,r} = \frac{2}{3}\mu_0 M\cos\theta$

$H^{\perp}_{in} = H_{in,r} = \vec{H}_{in}\cdot\hat{r} = -\frac{1}{3}M\cos\theta$

$\vec{K}_b = \vec{m}\times\hat{n} = m\hat{z}\times\hat{r} = M\sin\theta\,\hat{\phi}$

$B^{\parallel}_{in} = B_{in,\theta}\,\hat{\theta} + B_{in,\phi}\,\hat{\phi} = (\frac{2}{3}\mu_0\vec{m}\cdot\hat{\theta})\hat{\theta} + (\frac{2}{3}\mu_0\vec{m}\cdot\hat{\phi})\hat{\phi}$

$= \frac{2}{3}\mu_0 M \overset{\cos}{\cancel{\sin}}(\theta+\frac{\pi}{2})\hat{\theta} + 0$

$= -\frac{2}{3}\mu_0 m \overset{\sin}{\cancel{\cos}}\theta\ \hat{\theta}$

$\Rightarrow \vec{B}^{\parallel}_{out} = \vec{B}^{\parallel}_{in} + \mu_0(\vec{K}_b\times\hat{n})$

$= -\frac{2}{3}\mu_0 M \overset{\sin}{\cancel{\cos}}\theta\ \hat{\theta} + \mu_0 M\sin\theta\ \hat{\phi}\times\hat{r}$

$= +\frac{1}{3}\mu_0 M\sin\theta\ \hat{\theta}$

$\Rightarrow \vec{B}_{out} = \frac{\mu_0 M}{3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$ at surface of sphere

b) $\vec{A}_{dip} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$ $\qquad \vec{B} = \nabla\times\vec{A}$

$\vec{m}\times\hat{r} = m\hat{z}\times\hat{r} = m\sin\theta\,\hat{\phi}$

$\nabla\times\left(\frac{\sin\theta}{r^2}\hat{\phi}\right) = \hat{r}\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin^2\theta}{r^2}\right)\right] + \hat{\theta}\left[\frac{1}{r}\left(-\frac{\partial}{\partial r}\right)\left(\frac{\sin\theta}{r}\right)\right]$

$$= \hat{r}\left[\frac{1}{r\sin\theta}\ \frac{2\sin\theta\cos\theta}{r^2}\right] + \hat{\theta}\left[\frac{1}{r}\ \frac{\sin\theta}{r^2}\right]$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} m\left[2\cos\theta\ \frac{1}{r^3}\ \hat{r} + \sin\theta\ \frac{1}{r^3}\ \hat{\theta}\right]$$

$$\Rightarrow \vec{B}(R) = \frac{\mu_0 m}{4\pi R^3}\left[2\cos\theta\ \hat{r} + \sin\theta\ \hat{\theta}\right]$$

$\Rightarrow$ 2 fields same if $m = M \cdot \frac{4}{3}\pi R^3 = M \cdot$ volume

ok