A model of the water molecule H$_2$O is shown in MT Figure 9-B.

a. Where is the center of mass? $X = ?$, $Y = ?$, $Z = ?$  let $\theta = 52^o$

$$m_O = 16 \text{ amu}, \ m_H = 1 \text{ amu}$$

$$x_H = a \cos \theta, \ x_O = 0, \ X = \frac{m_Hx_H + m_Hx_H + m_Ox_O}{2m_H+m_O} = \frac{2}{18} a \cos \theta$$

$$y_H = \pm a \sin \theta, \ y_O = 0, \ Y = \frac{m_Hy_H - m_Hy_H + m_Oy_O}{2m_H+m_O} = 0$$

$z_H = 0$, $z_O = 0$, $Z = 0$. $Z$ and $Y$ can be seen directly from symmetry

b. What is the moment of inertia about the $x$ axis? $I_x = ?$

$$I_x = 2m_Hy_H^2 + 2m_Hz_H^2 + m_Oy_O^2 + m_Ox_O^2 = 2 \text{ amu} \ a^2 \sin^2 \theta$$

c. What is the moment of inertia about the $y$ axis? $I_y = ?$

$$I_x = 2m_Hx_H^2 + 2m_Hz_H^2 + m_Ox_O^2 + m_Ox_O^2 = 2 \text{ amu} \ a^2 \cos^2 \theta$$

d. What is the moment of inertia about the $z$ axis? $I_z = ?$

$$I_x = 2m_Hx_H^2 + 2m_Hy_H^2 + m_Ox_O^2 + m_Oy_O^2 = 2 \text{ amu} \ a^2(\sin^2 \theta + \cos^2 \theta) = 2 \text{ amu} \ a^2$$

e. What is the moment of inertia about an axis that goes through the hydrogen atoms? $I' = ?$ $x' = x - x_H$, $y' = y$, $z' = z$

$$I' = m_Ox'O^2 + 0 + 0 + 0 + 0 + 0 = (8 \text{ amu}) (a \cos \theta)^2$$

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