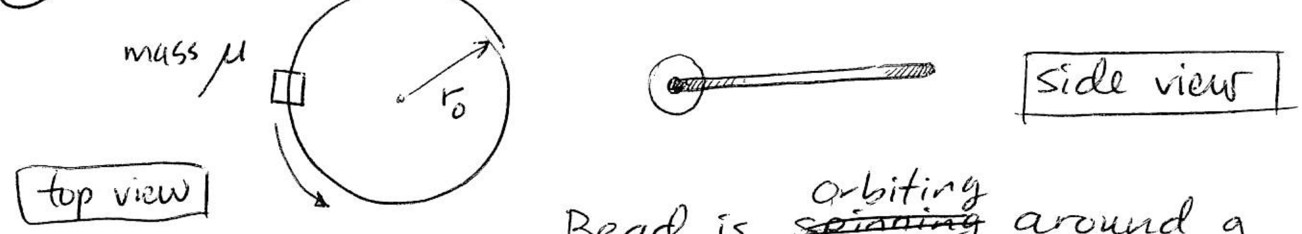
Practice Questions for final. (Classical)

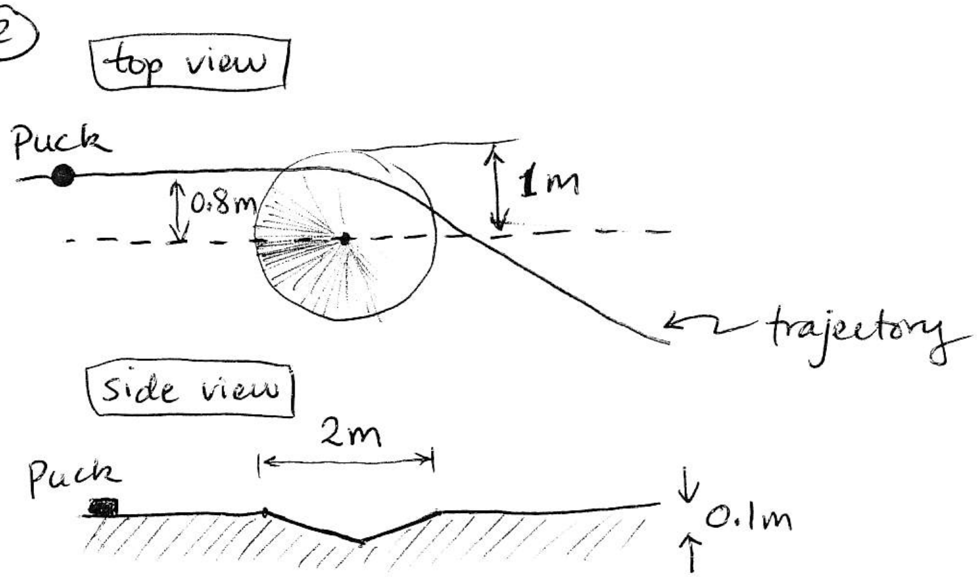
(1)



Bead is ~~spinning~~ orbiting around a frictionless wire track.

Find a relationship between the bead's kinetic energy and its orbital angular momentum about the center of the wire track.

(2)

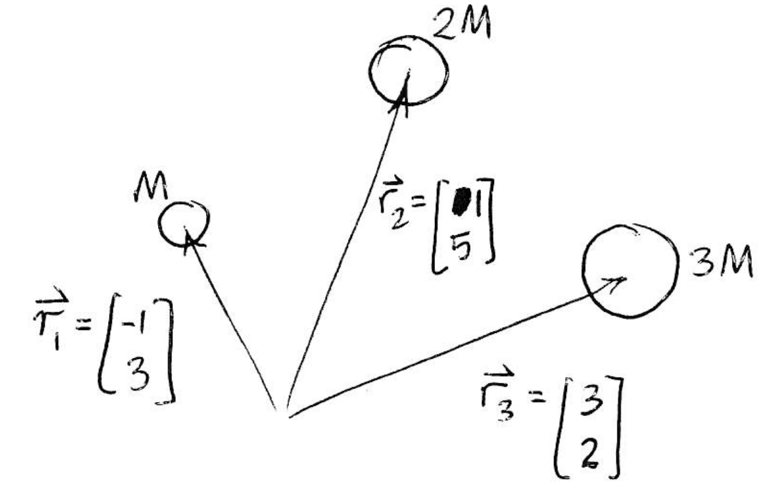


A hockey puck is sliding over the ice towards a cone-shaped hole in the ice.

At the center of the cone-shaped hole the depth is 0.1m.

How close will the puck come to the center of the hole? (Give numerical answer to within 1 s.f.)

(3)



Find the center of mass.

(4) Consider the earth orbiting the sun as usual. Suddenly the sun doubles its mass.

a) Sketch $U_{eff}(r)$ before & after the sun suddenly doubles its mass. Plot both curves on the same graph. Draw a horizontal line indicating the total energy (K.E. + P.E.) of the earth before and after the event.

b) Sketch the earth's trajectory around the sun for the year before ~~the~~ M_{sun} doubles & the year after M_{sun} doubles.

(5) Sketch $r(\phi) = \dfrac{2}{1 + 0.5\cos\phi}$ where r & ϕ are polar coordinates.

Label x-axis & y-axis intercepts with numerical values.

(6) Consider the trajectories of particles when they are subject to a $\frac{1}{r^2}$ central force.

Assume P.E. $\rightarrow 0$ as $r \rightarrow \infty$.

What are the conditions on K.E. & P.E. that ensure

(i) A parabolic trajectory

(ii) A hyperbolic trajectory

(iii) A circular trajectory

Practice Questions for final (Quantum)

① What is the ground state energy of positronium?
(i.e. An electron bound to a positron).

②

$$Y_1^1(\theta,\phi) = N \sin\theta \, e^{i\phi}$$

$$\hat{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

Use the functional forms shown above to ~~[illegible]~~ show that Y_1^1 is an eigen fn of $\hat{L}^2$?
~~What is~~ Identify the eigenvalue in your calculation.

③ A particle on a ring is prepared in the initial state

$$|\psi\rangle = \sqrt{\frac{1}{5}}\,|2\rangle - i\sqrt{\frac{4}{5}}\,|-1\rangle$$

Find the probability density as a function of time.

④ Show that the wavefns representing the $|100\rangle$ and $|210\rangle$ state are orthogonal.

(5) Consider a particle of mass m bound in an infinite square potential well in 3 dimensions.

$$V(x,y,z) = \begin{cases} 0 & \begin{array}{l} 0 \leq x \leq L \\ 0 \leq y \leq L \\ 0 \leq z \leq L \end{array} \\ \infty & \text{otherwise} \end{cases}$$

Use separation of variables in Cartesian coordinates to find the energy eigenvalues and eigenstates of this particle in a cubic box.

(6) Legendre polynomials can be found using the recursion relation

$$a_{n+2} = \frac{n(n+1) - \ell(\ell+1)}{(n+2)(n+1)} a_n$$

Find the Legendre polynomial associated with $\ell = 3$.

(7) Consider the diff. eqn $\frac{d^2y}{dt^2} + y = 0$

The solution can be written as a power series $y = \sum_{j=0}^{\infty} c_j y^j$. Find the recursion relation relating c_{j+2} to c_{j+1} and/or c_j

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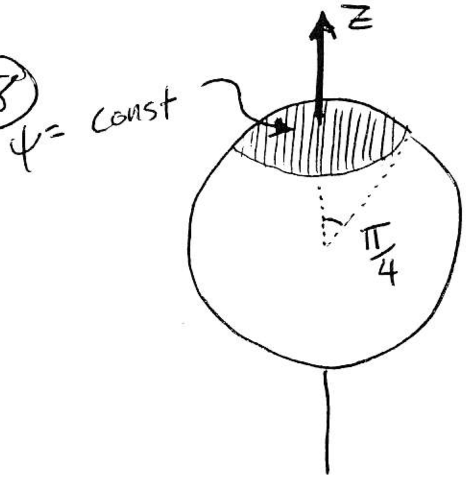


Consider an electron on a spherical surface.

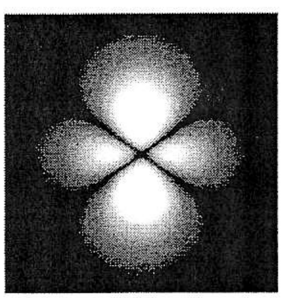
The initial wavestate is constant on the Northern cap and zero everywhere else.

a) Find the probability of measuring $L^2 = 2\hbar^2$

b) Will the probability change over time.

c) Will the probability density for finding the electron in a particular location ~~[illegible]~~ change over time?

9. The grayscale plot below shows the probability density on the x-z plane for an electron orbiting a proton. The electron is in an eigenstate of H, L^2 and L_z. What are the quantum numbers *n*, *l* and *m*? Explain how you figured it out.



10. Calculate the probability that an electron in the ground state of a hydrogen atom is located inside the nucleus of the atom. The nucleus is a single proton with spherical shape and radius 10^{-15} m.

Do the same calculation for uranium. i.e. The nucleus contains 92 protons plus 146 neutrons, and has a radius of 10^{-14} m. (For this calculation, ignore all but 1 of the electrons orbiting the uranium nucleus).